Pan Pearl River Delta Physics Olympiad 2016
2016 年泛珠三角及中华名校物理奥林匹克邀请赛
Sponsored by Institute for Advanced Study, HKUST
香港科技大学高等研究院赞助
Simplified Chinese Part-1 (Total 5 Problems) 简体版卷-1（共5题）
(9:00 am – 12:00 pm, 18 February, 2016)

Please fill in your final answers to all problems on the **summary sheet**.
请在**总答案纸**上填上各题的最后答案。
There are 5 problems. Please answer each problem using a **new sheet**.
卷一 5 题，每答 1 题，须采用**新一张纸**。
Please answer on each sheet using a **single column**. Do not use two columns on a single sheet.
每页纸请用**单一直列**的方式答题。不可以在一页纸上以双直列方式答题。
Please answer on **only one side** of each sheet. Do not use both sides of the sheet.
每张纸**单面**作答。不可以双面作答。
At the end of the competition, please arrange the **summary sheet as the first page**, followed by the answers to each problem in sequential order of the problems. If your answer to a problem requires more than one sheet, please arrange the sheets of the same problem in sequential order of the parts.
比赛结束时，请将**总答案纸放在首页**，随后把答题纸按题目次序排好。若答一题超过一页，请按分部次序排好答题纸。

1. Electrostatic Force (4 marks) 静电力 （4 分）

Consider a 2017-side regular polygon. There are 2016 point charges, each with charge q and located at a vertex of the polygon. Another point charge Q is located at the center of the polygon. The distance from the center of the regular polygon to its vertices is a. Find the force experienced by Q.
考虑一 2017-边正多边形。其中 2016 个角上各有一点电荷 q。 另有一个点电荷 Q 位于多边形的中心。 中心到每一个角的距离为 a。 求 Q 所受的力。

2. Capacitors (13 marks)

(a-c) Consider two clusters of electric charges. Cluster A consists of N charges q_1, q_2, …, q_N, located at positions $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N$ respectively. Cluster B consists of M charges $q'_1, q'_2, \ldots, q'_M$, located at positions $\vec{r}'_1, \vec{r}'_2, \ldots, \vec{r}'_M$ respectively.
(a-c) 考虑两组电荷。组 A 由 N 个电荷 q_1, q_2, …, q_N 组成, 并分别位于位置$\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N$。组 B 由 M 个电荷$q'_1, q'_2, \ldots, q'_M$组成, 并分别位于位置$\vec{r}'_1, \vec{r}'_2, \ldots, \vec{r}'_M$。

(a) Write the electric potential $\phi_A(\vec{r})$ at position $\vec{r}$ due to the charges in cluster A. (1 mark)
写下于位置$\vec{r}$ 由组 A 电荷形成的电势$\phi_A(\vec{r})$。（1 分）

(b) Write the electric potential energy $E_{B|A}$of cluster B due to the electric potential ϕ_A. (1 mark)
写下组 B 电荷因电势ϕ_A产生的电势能 $E_{B|A}$。 (1 分)

(c) What is the relation between $E_{B|A}$ and $E_{A|B}$? (1 mark)

$E_{B|A}$ 和 $E_{A|B}$有何关系?（1 分）

(d) Consider two large conducting plates as shown in Fig. 1a. The upper plate carries a uniform surface charge density σ' and the lower plate is grounded. Find the surface charge density of the lower plate and the potential $\phi'(z)$, where z is the height of an arbitrary location from the lower plate. (5 marks)
考虑如图 1a 所示两块很大的电导板。上板带有均匀面电荷密度σ', 而下板则接地。求下板的面电荷密度和电势$\phi'(z)$, 其中z为任意一点距离下板的高度。（5 分）

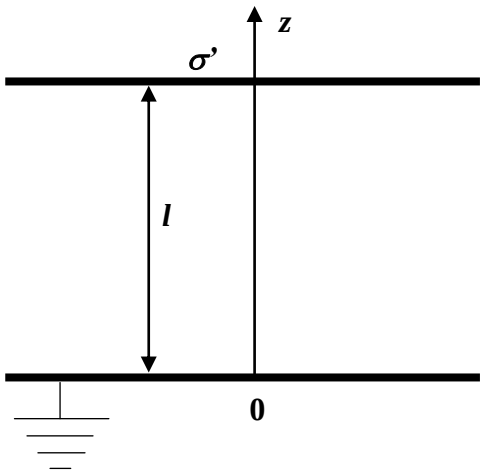

Figure 1a 图 1 a

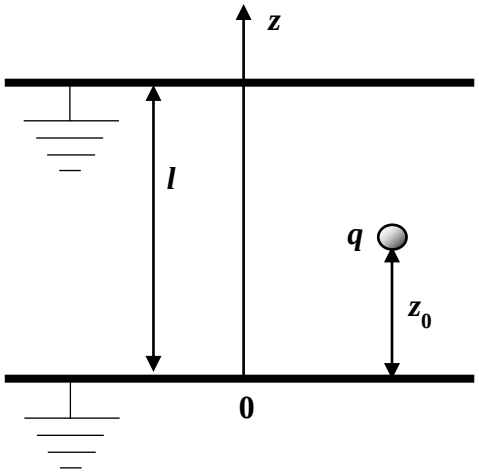

Figure 1b 图 1b

(e) A point charge q is placed between two very large grounded parallel conducting plates. If z_0 is the distance between q and the lower plate, find the total charge induced on the upper plate in terms of q, z_0, and l, where l is the distance between the plates, as shown in Fig. 1b. (5 marks)
如图 1b 所示, 在相距为 l 的两块平行大电导板间放置电荷 q, 其到下板的距离为 z_0。求上板的总感应电荷。以 q、z_0 和 l 表示你的答案。（5 分）

3. Cannonballs and Bombs (10 marks) 砲彈和炸彈 （10 分）

(a) Envelope of safety: A ground based cannon can fire a cannonball at a fixed speed of u in any direction. The envelope of safety is the curve inside which a target can be hit by the cannonball, and outside which there is no possibility of a target getting hit by the cannonball. Find the equation of the envelope of safety in space. (3 marks)
安全区域边界: 一门位于地面的大炮能以固定速率 u 向任何方向发射炮弹。 若目标在安全区域边界内, 则有可能被炮弹打中。 若在其外, 则不可能被炮弹打中。 求在空中的安全区域边界的方程式。 （3 分）

(b) A bomb explodes at a height of H into many small fragments. It is given that after the explosion the fragments have the same speed u and a uniform angular distribution in all directions. After some time all fragments hit the ground and the collisions with the ground are perfectly inelastic. Find the radius R of the distribution of the debris. (2 marks)

一个炸弹在高度 H 处爆炸成很多小碎片。 已知刚爆炸后各碎片以同样的速率 u 和均匀的角分布向各个方向散开。 其后各碎片都坠到地面上。 假设所有碎片与地面的碰撞皆为完全非弹性碰撞。 求炸弹残骸的分布半径 R。 (2 分)

(c) A bomb explodes on the ground. Its fragments are projected at the same speed u, and the angular distribution is uniform within a narrow angle α with the upward vertical direction. After some time all fragments hit the ground. Let the mass of the bomb be M. Find the radius R of the distribution of the debris up to order α. Calculate the radial density distribution $\rho(r)$ within radius R up to order r^2, where $\rho(r)2\pi r dr$ is the mass of the debris located at a distance r to $r + dr$ from the centre of the distribution. (5 marks)

[Remark: $\tan\varepsilon \approx \varepsilon\left(1+\frac{\varepsilon^2}{3}\right)$ and $\sin\varepsilon \approx \varepsilon\left(1-\frac{\varepsilon^2}{6}\right)$ for $\varepsilon \ll 1$.]

一个炸弹在地面爆炸。爆炸后各碎片以同样速率 u 射出，角度分布则限在与垂直向上方向的狭小夹角α内，而在这范围内角度分布均匀。其后各碎片都坠到地面上。设炸弹的质量为 M。求炸弹残骸的分布半径 R, 准确至α 的第一阶。定义径密度分布$\rho(r)$，使得$\rho(r)2\pi r dr$为距离残骸中心 r 至 r+dr 范围内的残骸质量。求半径 R 内的$\rho(r)$, 准确至 r 的第二阶。（5 分）

[注: 当 ε ≪1 时, $\tan\varepsilon \approx \varepsilon\left(1+\frac{\varepsilon^2}{3}\right)$及$\sin\varepsilon \approx \varepsilon\left(1-\frac{\varepsilon^2}{6}\right)$。]

4. Collisions (14 marks) 碰撞 (14 分)

A thin rod with length L, mass m and uniform density lies on the y-axis with its midpoint at the origin. A point object A with mass m travels with velocity u in the positive x direction hits the rod with impact parameter h, where $-L/2 \le h < L/2$, as shown in Fig. 2a. The collision is perfectly inelastic.

如图 2a 所示, 一根长度为 L、质量为 m、密度均匀的幼棒处在 y 轴上。 棒的中心点位于原点。 一质量为 m 的质点 A 以速度 u 向正 x 方向运动, 并以碰撞参数 h 与棒碰撞, 其中$-L/2 \le h < L/2$,。碰撞为完全非弹性碰撞。

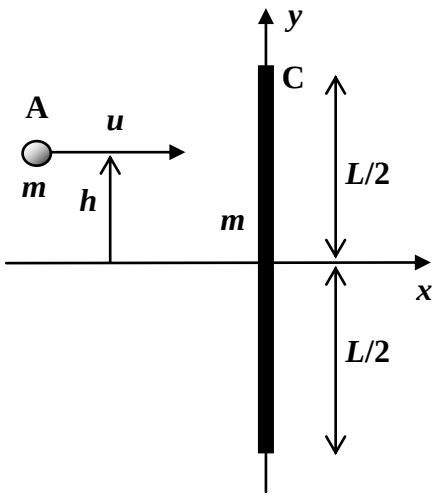

Figure 2a 图 2a

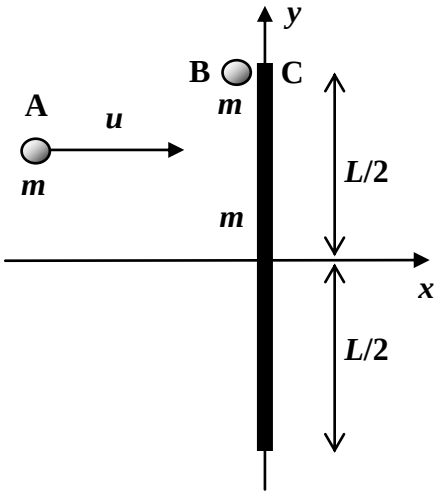

Figure 2b 图 2b

(a) Find the total kinetic energy just after the collision between A and the rod. (6 marks)
求 A 刚与棒碰撞后系统的总动能。（6 分）

(b) Find the velocity v of point C at the top end of the rod as a function of h. (2 marks)
求棒上端点 C 的速度 v 与 h 的函数关系。（2 分）

(c) Find H such that $v(H) = 0$. (1 mark)
求 H 使得 $v(H) = 0$。（1 分）

(d) Suppose another point object B of mass m is located very close to point C, at the left hand side, as shown in Fig. 2b. Further suppose the point object A hits the rod at the lower end. Find the velocity of the point object B just after the rod collides elastically with it. (5 marks)
假设另一质量为 m 的质点 B 的位置与棒顶端 C 的左边非常接近，如图 2b 所示。再设点 A 撞到棒的下端。求棒与质点 B 产生完全弹性碰撞后，质点 B 的速度。（5 分）

5. Thermodynamic Cycle (9 marks) 热力学循环 (9 分)

Consider the thermodynamic cycle of an ideal monatomic gas shown in the pV diagram in Fig. 3. The cycle consists of four processes:

A → B: Isobaric expansion at pressure rp, where $r > 1$
B → C: Isothermal expansion at temperature T_2
C → D: Isobaric compression at pressure p
D → A: Isothermal compression at temperature T_1

考虑图 3 中所示一种单原子理想气体的热力学循环的 pV 图。 该循环包括四个过程:

A → B: 压强 rp 下的等压膨胀, 其中 $r > 1$
B → C: 温度 T_2 下的等温膨胀
C → D: 压强 p 下的等压压缩
D → A: 温度 T_1 下的等温压缩

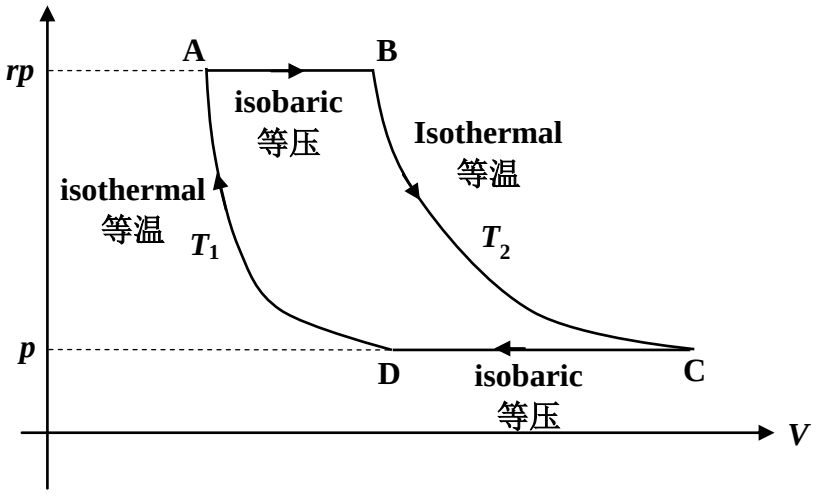

Figure 3 图 3

(a) Write the highest temperature T_H and lowest temperature T_C in the cycle. No proof is required. (1 mark)
无须证明, 写下循环中的最高温度 T_H 和最低温度 T_C。（1 分）

(b) Write the efficiency e_C of a Carnot engine operating with a hot reservoir at temperature T_H and a cold reservoir at temperature T_C. (1 mark)
一卡诺热机在温度为 T_H 的高温热库和温度为 T_C 的低温热库间运行。 写下其热效率 e_C。(1 分)

(c) Given that the gas is in thermal contact with a hot reservoir with temperature T_H whenever heat is added to the gas, and in thermal contact with a cold reservoir with temperature T_C whenever heat is removed from the gas, find the efficiency e of an engine running the cycle in the pV diagram. Express your answer in terms of T_C, T_H, p, and r. (5 marks)
已知一热机在循环运行中，气体吸热时永远与温度为 T_H 的热库处于热接触, 气体放热时永远与温度为 T_C 的热库处于热接触。求其热效率 e。 以 T_C、T_H、p 和 r 表示你的答案。（5 分）

(d) Find the ratio $\frac{e}{e_C}$. Hence suggest a parameter regime in which the efficiency approaches that of the ideal engine. (2 marks)
求比例 $\frac{e}{e_C}$。根据答案, 提出能使热效率趋近理想热机热效率的参数范围。 (2 分)

《THE END 完》