

[Marking Scheme]

Theoretical Question 1

When will the Moon become a Synchronous Satellite?

(1)	3.0	0.4	location of center of mass C ➤ 0.2 for distance to Earth ➤ 0.2 for distance to Moon
		0.4	Orbital angular momentum of the Moon ℓ_M ➤ 0.2 for formula ➤ 0.2 for numerical value
		0.4	spin angular momentum of the Moon S_M ➤ 0.2 for formula ➤ 0.2 for numerical value
		0.4	Orbital angular momentum of the Earth ℓ_E ➤ 0.2 for formula ➤ 0.2 for numerical value
		0.4	spin angular momentum of the Earth S_E ➤ 0.2 for formula ➤ 0.2 for numerical value
		0.2	knowing total angular momentum of a system is the <i>sum</i> of orbital and spin angular momenta
		0.8	total angular momentum of the Earth-Moon system L ➤ 0.2 for order of magnitude ➤ 0.4 for value to two significant digits ➤ 0.2 for unit
(2)	3.0	0.8	Newton's form of Kepler's third law $\omega^2 r^3 = G(M_E + M)$ ➤ 0.6 for $\omega^2 r^3 = \text{constant}$ ➤ 0.2 for expression of constant $G(M_E + M)$
		0.4	realizing total orbital angular momentum $\ell = \ell_E + \ell_M$ is a function of ω or r alone
		0.2	realizing spin angular momentum of the Moon is negligible
		0.4	resorting to the law of conservation of total angular momentum
		1.2	period of rotation of the Earth T ➤ 0.3 for order of magnitude in units of second ➤ 0.4 for value to two significant digits ➤ 0.5 for providing an equation for finding T
(3)	4.0	0.2	0.2 for knowing $\Gamma = \left(\frac{r_0}{r}\right)^6 \Gamma_0$
		0.4	realizing relation between torque and rate of slowdown of Earth's rotation: $dS_E / dt = \Gamma$
		0.4	concluding $-\Gamma$ is equal to rate of increase of total orbital angular momentum of the Earth-Moon system: $d\ell / dt = -\Gamma$
		1.0	current value of the torque Γ_0 ➤ 0.2 for realizing ℓ is related to r ➤ 0.3 for converting the derivative $d\ell / dt$ to dr / dt

	<ul style="list-style-type: none"> ➤ 0.4 for value of Γ_0 ➤ 0.1 for unit of torque
0.6	<p><i>converting the derivative $d\ell/dt$ to $d\omega/dt$</i></p> <ul style="list-style-type: none"> ➤ 0.1 for realizing ℓ is related to ω ➤ 0.1 for realizing Γ is related to ω ➤ 0.4 for converting $d\ell/dt = -\Gamma$ into a differential equation for ω
0.6	<p><i>providing an equation for the solution of t_f</i></p>
0.8	<p><i>estimate of time t_f</i></p> <ul style="list-style-type: none"> ➤ 0.3 for order of magnitude in units of second ➤ 0.4 for value to two significant digits ➤ 0.1 for value in years