Solution and Marking Scheme
Experiment

I. Determination of Capacitance

a) $$\overline{P} = I^2 R = \frac{\frac{1}{2}\mathcal{E}_0^{\,2}}{R^2 + \left(\frac{1}{\omega C}\right)^2} R$$ (1.0 point)

b) $$\frac{d}{dR}\overline{P} = 0$$ (0.3 point)

$$\frac{d}{dR}\overline{P} = \frac{d}{dR}\left(\frac{\frac{1}{2}\mathcal{E}_0^{\,2}}{R^2 + \left(\frac{1}{\omega C}\right)^2} R\right)$$ (0.4 point)

$$= \tfrac{1}{2}\mathcal{E}_0^{\,2}\,\frac{R^2 + \left(\frac{1}{\omega C}\right)^2 - R(2R)}{\left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)^2}$$

condition for $\overline{P}_{\max}$: $$R = \frac{1}{\omega C}$$ (0.3 point)

c. (1 point) $$\overline{P} = \frac{\frac{1}{2}\mathcal{E}_0^{\,2}}{R^2 + \left(\frac{1}{\omega C}\right)^2} R = \frac{\frac{1}{2}\mathcal{E}_0^{\,2} R}{R^2\left[1 + \left(\frac{1}{R\omega C}\right)^2\right]} = \frac{\frac{1}{2}\mathcal{E}_0^{\,2}}{R\left[1 + \left(\frac{1}{R\omega C}\right)^2\right]}$$

$$=> \frac{1}{R\overline{P}} = \frac{2}{\mathcal{E}_0^{\,2}}\left[1 + \frac{1}{R^2}\frac{1}{\omega^2 C^2}\right]$$

$$\frac{1}{R\overline{P}} = \frac{1}{V^2} = \frac{2}{\mathcal{E}^2{}_0} + \frac{2}{\mathcal{E}^2{}_0}\left(\frac{1}{\omega C}\right)^2 \frac{1}{R^2}$$

Note: The linear graph will be $\frac{1}{R\overline{P}}$ or $\frac{1}{V^2}$ versus $\frac{1}{R^2}$. If a is the slope and b is the intercept with the Y axis, then

$$\frac{1}{\omega^2 C^2} = \frac{a}{b} \quad \Rightarrow \quad C = \frac{1}{\omega}\sqrt{\frac{b}{a}}$$

An alternative method:

$$\frac{V^2}{R^2} = \frac{\frac{1}{2}\mathcal{E}_0^{\,2}}{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\frac{R^2}{V^2} = \left(R^2 + \left(\frac{1}{\omega C}\right)^2\right)\frac{2}{\mathcal{E}_0^{\,2}}$$

$$\frac{1}{V^2} = \left[1+\left(\frac{1}{\omega C}\right)^2\frac{1}{R^2}\right]\frac{2}{\mathcal{E}_0^{\,2}} = \frac{2}{\mathcal{E}_0^{\,2}} + \frac{2}{\mathcal{E}_0^{\,2}}\left(\frac{1}{\omega C}\right)^2\frac{1}{R^2}$$

$$R^2 = \left(\frac{1}{2}\mathcal{E}_0^{\,2}\right)\left(\frac{R}{V}\right)^2 - \left(\frac{1}{\omega C}\right)^2$$

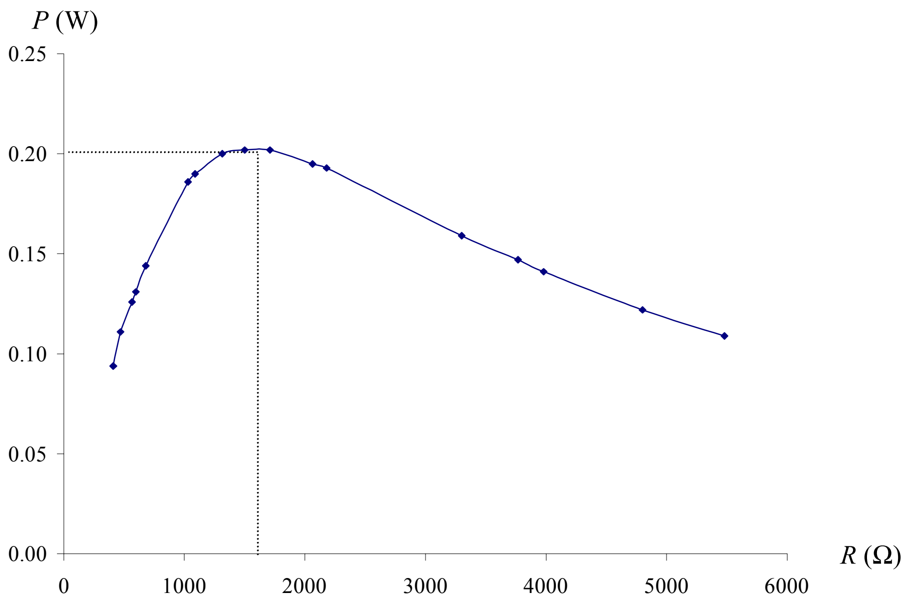
Note: The graph will be R^2 versus $\left(\frac{R}{V}\right)^2$ and C is determined from the Y-intercept.

d)

No.	Resistor(s)	R (Ω)	V (V)	$\overline{P} = \frac{V^2}{R}$ (W)
1	R_A	680	9.86	0.144
2	R_B	1500	17.36	0.202
3	R_C	3300	22.81	0.159
4	$R_A + R_B$	2180	20.49	0.193
5	$R_A // R_B$	468	7.28	0.111
6	$R_B + R_C$	4800	23.98	0.122
7	$R_B // R_C$	1032	13.78	0.186
8	$R_C + R_A$	3980	23.66	0.141
9	$R_C // R_A$	564	8.42	0.126
10	$R_A + R_B + R_C$	5480	24.40	0.109
11	$(R_A // R_B) + R_C$	3768	23.43	0.147
12	$(R_B // R_C) + R_A$	1712	18.63	0.202
13	$(R_C // R_A) + R_B$	2064	20.15	0.195
14	$(R_A // R_B) // R_C$	410	6.22	0.094
15	$(R_A + R_B) // R_C$	1313	16.18	0.200
16	$(R_B + R_C) // R_A$	596	8.82	0.131
17	$(R_C + R_A) // R_B$	1089	14.36	0.190

(2.5 points):(data points = 17; 2.5, >13; 2.0, >9; 1.5; >3; 1.0, ≤3; 0.5)

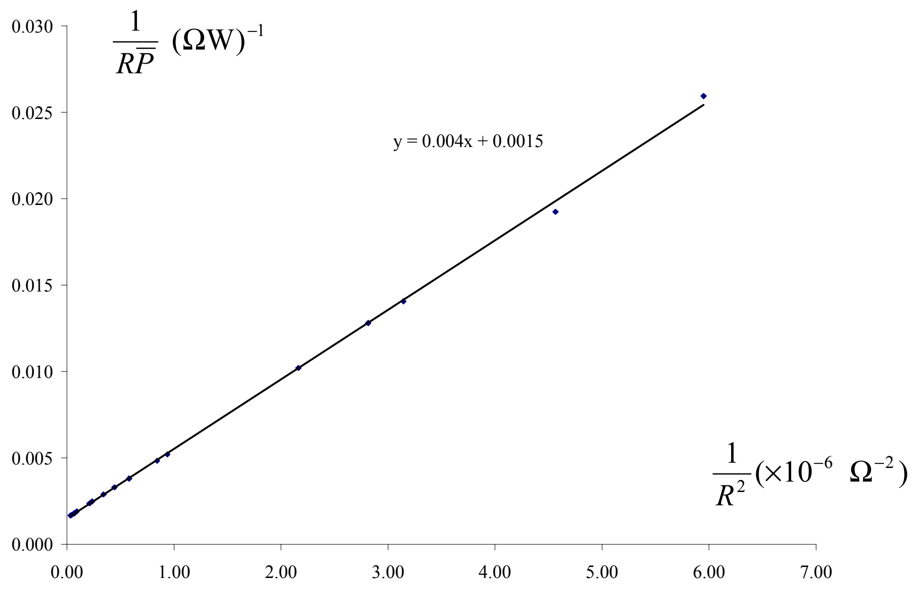
e) (1.5 points):good graph (0.5 point): correct value



$$R \text{ at } \overline{P}_{\max} = 1600\Omega \Rightarrow C = \frac{1}{\omega R} = \frac{1}{2\pi \times 50 \times 1600} = 1.9\times10^{-6}\,\text{F} = 1.9\mu\text{F}$$

f) Linear graph.

$R(\Omega)$	V (V)	$\overline{P} = \frac{V^2}{R}$ (W)	$\frac{1}{R\overline{P}}$ $(\Omega\text{W})^{-1}$	$\frac{1}{R^2}$ $(\times10^{-6}\,\Omega^{-2})$
410	6.22	0.094	0.0259	5.948
468	7.28	0.111	0.0193	4.565
564	8.42	0.126	0.0141	3.143
596	8.82	0.131	0.0128	2.815
680	9.86	0.144	0.0102	2.162
1032	13.78	0.186	0.0052	0.938
1089	14.36	0.190	0.0048	0.843
1313	16.18	0.200	0.0038	0.580
1500	17.36	0.202	0.0033	0.444
1712	18.63	0.202	0.0029	0.341
2064	20.15	0.195	0.0025	0.234
2180	20.49	0.193	0.0024	0.210
3300	22.81	0.159	0.0019	0.091
3768	23.43	0.147	0.0018	0.070
3980	23.66	0.141	0.0018	0.0631
4800	23.98	0.122	0.0017	0.0434
5480	24.40	0.109	0.0017	0.0333

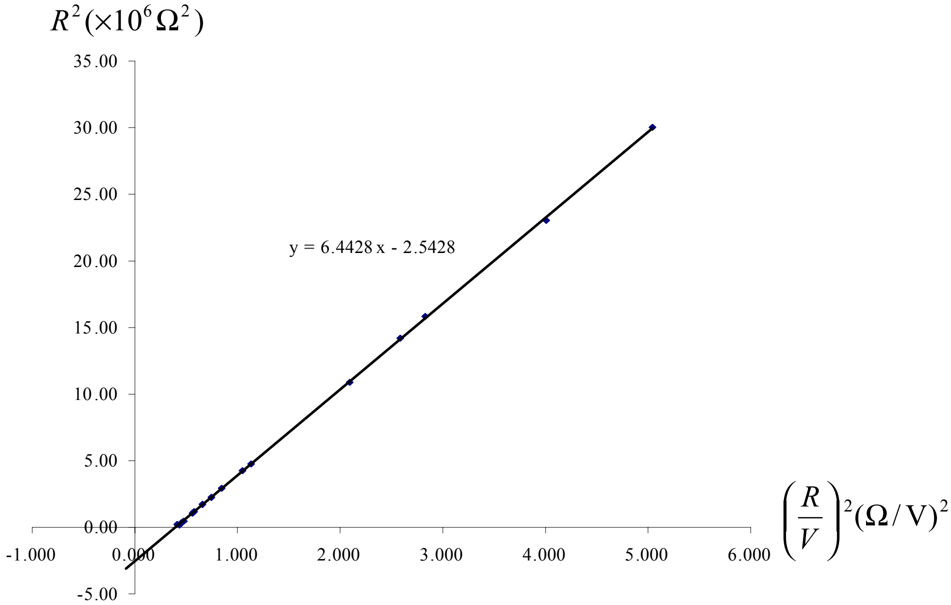


slope = $a = 0.004 \times 10^6\ \Omega$/W , Y-intercept = $b = 0.0015\ (\Omega\text{W})^{-1}$:

$$\frac{1}{\omega^2 C^2} = \frac{a}{b} \quad \Rightarrow \quad C = \frac{1}{\omega}\sqrt{\frac{b}{a}} = 1.95\times10^{-6}\ \text{F} = 1.95\,\mu\text{F}$$

An alternative method of linear graph

$R(\Omega)$	V(V)	$\overline{P}=\dfrac{V^2}{R}$ (W)	$\left(\dfrac{R}{V}\right)^2$ $(\Omega/\mathrm{V})^2$	R^2 $(\times 10^6\,\Omega^2)$
410	6.22	0.094	4345	0.17
468	7.28	0.111	4133	0.22
564	8.42	0.126	4487	0.32
596	8.82	0.131	4566	0.36
680	9.86	0.144	4756	0.46
1032	13.78	0.186	5609	1.07
1089	14.36	0.190	5751	1.19
1313	16.18	0.200	6585	1.72
1500	17.36	0.202	7466	2.25
1712	18.63	0.202	8445	2.93
2064	20.15	0.195	10492	4.26
2180	20.49	0.193	11320	4.75
3300	22.81	0.159	20930	10.89
3768	23.43	0.147	25863	14.20
3980	23.66	0.141	28297	15.84
4800	23.98	0.122	40067	23.04
5480	24.40	0.109	50441	30.03



Graphical analysis: Y-intercept $=\left(\dfrac{1}{\omega C}\right)^2 = 2.5428\times 10^6\ \Omega^2$

$$\frac{1}{\omega C} = 1.595\times 10^3\ \Omega \quad \Rightarrow \quad C = 1.99\times 10^{-6}\ \mathrm{F} = 1.99\,\mu\mathrm{F}$$

(1.5 points): good graph
(0.5 point): correct value

a) Estimation of the uncertainty in the values of C obtained in e)
(0.25 point)

Estimation of the uncertainty in the values of C obtained in f)
(0.25 point)