16th ASIAN PHYSICS OLYMPIAD 2015
3rd-11th MAY, HANGZHOU, CHINA

Experimental Competition

May 7, 2015

08:30-13:30 hours

Marking Scheme

Experiment A

A.1

Choose a PZT plate and use the Vernier caliper to measure its length *l*, width *w*, and thickness *t*. Use the electronic weighing scale to measure its mass *m*. Use the DMM and the Kelvin clip to measure its capacitance *C* (at ambient temperature).

Considering the slight non-uniformity in the dimensions of the PZT plate and the uncertainties of instrumental readings, repeat each measurement several times and then calculate the mean and the standard error.

	l (mm)	*w* (mm)	*t* (mm)	*m* (g)	*C* (nF)
1	45.00	6.98	1.00	2.24	18.19
2	45.02	7.00	1.00	2.23	18.13
3	45.00	7.02	0.98	2.26	18.17
4	45.02	7.04	0.98	2.25	18.19
5	45.02	7.04	1.00	2.25	18.20
6	45.00	7.04	1.00	2.25	18.21
Avg.	45.01±0.02	7.02±0.02	0.99±0.02	2.25±0.01	18.18±0.02

Alternative:

<table>
<tr><th></th><th>l (mm)</th><th>w(mm)</th><th>t(mm)</th><th>m(g)</th><th>C(nF)</th></tr>
<tr><td>1</td><td rowspan="6">45.00</td><td rowspan="6">7.02</td><td rowspan="6">1.00</td><td rowspan="6">2.25</td><td>18.19</td></tr>
<tr><td>2</td><td>18.13</td></tr>
<tr><td>3</td><td>18.17</td></tr>
<tr><td>4</td><td>18.19</td></tr>
<tr><td>5</td><td>18.20</td></tr>
<tr><td>6</td><td>18.21</td></tr>
<tr><td>Avg.</td><td>45.00±0.02</td><td>7.02±0.02</td><td>1.00±0.02</td><td>2.25±0.01</td><td>18.18±0.02</td></tr>
</table>

If repeated measurement of dimension and mass gives the same result.

Total:**1.6**

0.2 data table.

0.2 units.

-0.1 each unit missing.

0.3 standard error.

-0.1 each error missing.

0.3 correct number of significant figures.

-0.1 each wrong significant figure.

0.3 correct reading of Vernier caliper.

0.2 right value of *C*

(**0.2**:17.30~19.10nF)

(**0.1**:16.40~17.30nF, 19.10~20.00nF)

0 otherwise.

0.2 repeated measurement of capacitance

(**0.2**: repeat≥6times)

(**0.1**: repeat≥3times)

0 otherwise.

A.2	Now calculate the density ρ and the relative permittivity ε_r of the PZT plate. Based on standard errors obtained from A.1, carry out the error analysis to estimate the uncertainties of ρ and ε_r (vacuum permittivity ε_0 =8.85×10^{-12} F/m).	Total:**1.4**
	$$\rho = \frac{m}{lwt} = 7.20\times10^3 \ \mathrm{kg/m^3}$$	**0.1** unit of ρ. **0.2** right value of ρ. (**0.2**:6.85~7.55) (**0.1**:6.50~6.85,7.55~7.90) **0** otherwise.
	$$\frac{\Delta\rho}{\rho} = \sqrt{(\frac{\Delta m}{m})^2 + (\frac{\Delta l}{l})^2 + (\frac{\Delta w}{w})^2 + (\frac{\Delta t}{t})^2} = 0.021$$	**0.2** $\frac{\Delta\rho}{\rho}$ expression.
	$$\therefore\ \Delta\rho = 0.15\times10^3 \ \mathrm{kg/m^3}$$ $$\rho = (7.20\pm0.15)\times10^3 \ \mathrm{kg/m^3}$$	**0.2** right value of Δρ. (**0.2**:0.05~0.30) (**0.1**:0.01~0.05, 0.30~0.50) **0** otherwise.
	$$\varepsilon_r = \frac{Ct}{\varepsilon_0 lw} = 6.44\times10^3$$ $$\frac{\Delta\varepsilon_r}{\varepsilon_r} = \sqrt{(\frac{\Delta C}{C})^2 + (\frac{\Delta l}{l})^2 + (\frac{\Delta w}{w})^2 + (\frac{\Delta t}{t})^2} = 0.021$$	The same criteria for ε_r.
	$$\Delta\varepsilon_r = 0.14\times10^3$$ $$\varepsilon_r = (6.44\pm0.14)\times10^3$$	02.right value of ε_r (**0.2**:6.10~6.80) (**0.1**:5.75~6.10, 6.80~7.15) **0** otherwise
	Alternative: calculate ρ each time:7.13,7.08,7.30,7.24,7.10,7.10, $$\sigma_{\bar\rho} = \sqrt{\frac{\sum_{i=1}^{n}\left(\rho_i - \bar\rho\right)^2}{n(n-1)}} = 0.04\times10^3 \ \mathrm{kg/m^3}$$ $$\rho = (7.16\pm0.04)\times10^3 \ \mathrm{kg/m^3}$$	**0.2** right value of $\Delta\varepsilon_r$. (**0.2**:0.05~0.30) (**0.1**:0.01~0.05, 0.30~0.50) **0** otherwise.
	Or $$\sigma_{\bar\rho} = \sqrt{\frac{\sum_{i=1}^{n}\left(\rho_i - \bar\rho\right)^2}{(n-1)}} = 0.10\times10^3 \ \mathrm{kg/m^3}$$ $$\rho = (7.16\pm0.10)\times10^3 \ \mathrm{kg/m^3}$$	---------------- Alternative: **0.2** ρ. **0.1** unit of ρ. **0.1** Δρ expression. **0.1** right value of Δρ. (**0.1**:0.01~0.40) **0** otherwise. **0.2** data points.

		(**0.2** measure≥6times) (**0.1**:3~5times) **0** otherwise. The same criteria for ε_r.

Experiment B

B.1	Derive the expressions for the resonant frequency f_r and the antiresonant frequency f_a of the equivalent circuit. The impedance of the capacitance C_0, C_1 and inductance L_1 are $$Z_0=\frac{1}{i\omega C_0},\quad Z_1=\frac{1}{i\omega C_1},\quad Z_2=i\omega L_1 \qquad (1)$$ Respectively. Assume the total impedance of the equivalent circuit is Z, then we have $$\frac{1}{Z}=\frac{1}{Z_0}+\frac{1}{Z_1+Z_2}=i\omega C_0+\frac{1}{\frac{1}{i\omega C_1}+i\omega L_1}=i\omega\frac{C_0-\omega^2 L_1C_0C_1+C_1}{1-\omega^2L_1C_1}$$ (2) Resonance condition: $$1-\omega^2L_1C_1=0\Rightarrow f_r=\frac{1}{2\pi\sqrt{L_1C_1}} \qquad (3)$$ Antiresonance condition: $$C_0-\omega^2L_1C_0C_1+C_1=0\Rightarrow f_a=\frac{1}{2\pi}\sqrt{\frac{1}{L_1C_1}+\frac{1}{L_1C_0}} \qquad (4)$$	Total:**1.0** **0.1** C_0 impedance. **0.1** C_1 impedance. **0.1** L_1 impedance. **0.3** total impedance. **0.2** resonant frequency. **0.2** antiresonant frequency.

B.2

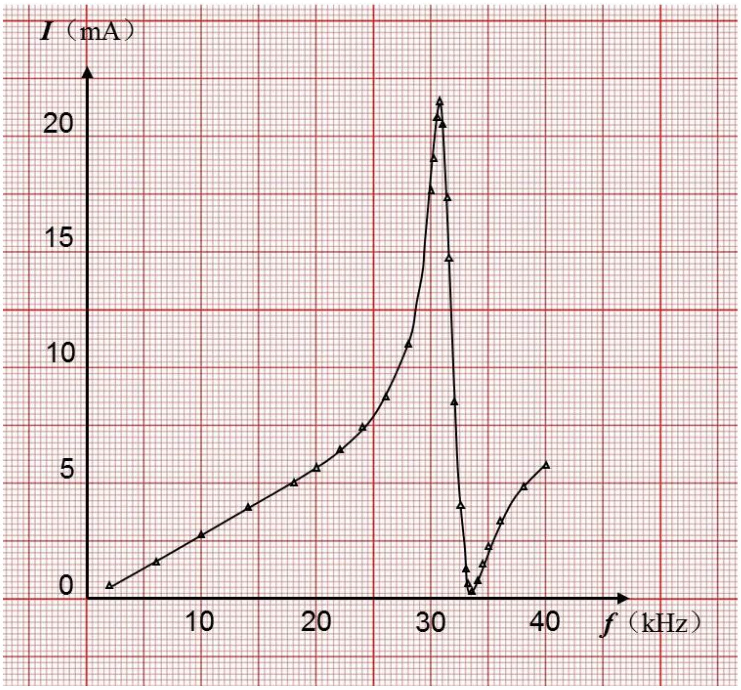
Measure the AC current I through the PZT plate as a function of the signal frequency f. Draw the I-f curve and find the resonant frequency f_r and the antiresonant frequency f_a. Calculate the piezoelectric coefficient d accordingly.

Freq (kHz)	I(mA)	Freq (kHz)	I(mA)	Freq (kHz)	I(mA)	Freq (kHz)	I(mA)
2	0.58	30.5	20.86	32.4	4.82	34.3	1.18
4	1.13	30.6	21.28	32.5	4.06	34.0	1.37
6	1.68	30.7	21.50	32.6	3.37	34.5	1.52
8	2.21	30.8	21.47	32.7	2.76	34.6	1.67
10	2.75	30.9	21.13	32.8	2.20	34.7	1.82
12	3.29	31.0	20.51	32.9	1.73	34.8	1.96
14	3.85	31.1	19.64	33.0	1.29	34.9	2.10
16	4.42	31.2	18.55	33.1	0.94	35	2.23
18	5.03	31.3	17.35	33.2	0.66	36	3.35
20	5.69	31.4	16.06	33.3	0.47	37	4.18
22	6.46	31.5	14.73	33.4	0.36	38	4.82
24	7.39	31.6	13.40	33.5	0.31	39	5.34
26	8.72	31.7	12.10	33.6	0.33	40	5.78
28	11.05	31.8	10.85	33.7	0.38		
30	17.69	31.9	9.65	33.8	0.48		
30.1	18.34	32.0	8.55	33.9	0.60		
30.2	18.99	32.1	7.50	34.0	0.74		
30.3	19.66	32.2	6.52	34.1	0.89		
30.4	20.29	32.3	5.63	34.2	1.04		

Total:**3.5**
0.2 data table.
0.2 units.
0.3 significant figures.
0.3 ≥10 data points.
0.3 *fr*.
0.3 *fa*.
0.3 0.1kHz freq. resolution near *fr* and *fa*.

0.5 figure
(**0.1:** data points,
0.1: units,
0.1: axis label,
0.1: axis ticks label,
0.1: smooth curve).

0.1: unit of *d*.
1.0 right value of *d*
(**1.0**:4.20~4.70)
(**0.5**:3.95~4.20, 4.70~4.95)
0 otherwise.



f_r=30.7 kHz, f_a=33.5 kHz

$$d=\sqrt{\frac{\varepsilon_0\varepsilon_r}{128f_r^4l^2\rho[\frac{1}{(2\pi f_a)^2-(2\pi f_r)^2}+\frac{1}{32f_r^2}]}}=4.44\times10^{-10}\ \text{m/V (or C/N)}$$

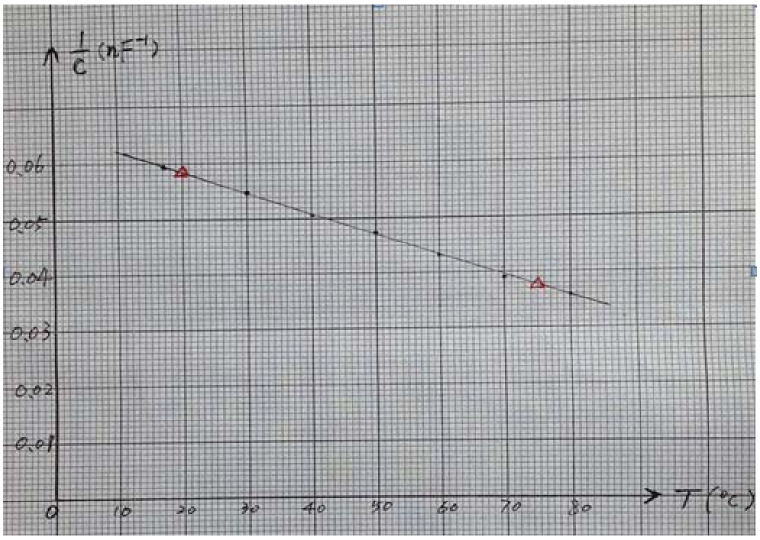
Experiment C

<table>
<tr>
<td>C.1</td>
<td>Now measure the capacitance of the PZT plate at various temperatures and record the data.

<table>
<tr><td>T(°C)</td><td>17.0</td><td>30.0</td><td>40.0</td><td>50.0</td><td>60.0</td><td>70.0</td><td>80.0</td></tr>
<tr><td>C(nF)</td><td>16.80</td><td>18.25</td><td>19.77</td><td>21.08</td><td>23.07</td><td>25.60</td><td>27.80</td></tr>
<tr><td>1/C(nF^{-1})</td><td>0.0595</td><td>0.0548</td><td>0.0506</td><td>0.0474</td><td>0.0433</td><td>0.0391</td><td>0.0360</td></tr>
</table>
</td>
<td>Total:1.5
0.2 data table.
0.2 units.
0.2 significant figures.
0.4 data points
(0.4: ≥6 data points)
(0.2: 4~5 data points)
0 otherwise.

0.5 temperature range
(0.5: from room temperature to ≥80℃)
(0.3: temperature range 35~50℃)
(0.1: temperature range 20~35℃)
0 otherwise.</td>
</tr>
</table>

<table>
<tr>
<td>C.2</td>
<td>Analyze the data, draw a proper plot and calculate the Curie temperature accordingly.



From the straight line choose two points P1(30.0,0.05275), P2(90.0,0.03075) to calculate the slope

$$k = \frac{\Delta y}{\Delta x} = -0.000367$$

We can get the linear function

$$y = -0.000367x + 0.0638$$

Let y=0, then we can get the Curie temperature

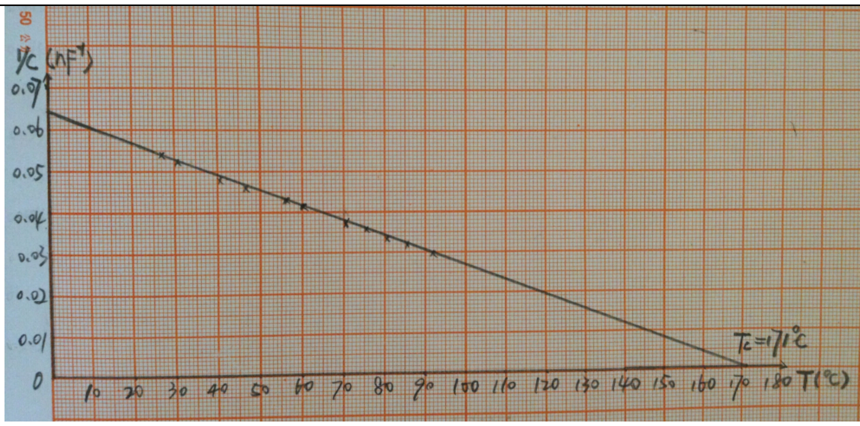
$$T_c = 174^\circ C$$

Alternative method: extent the straight line to intercept with the x axis
to get the Curie temperature.</td>
<td>Total:2.5
0.1 axis label
0.1 axis unit.
0.1 axis tick label.
0.3 Choice of Scale (to cover 70% or more space on graph sheet).
0.2 data marker.
0.2 straight line.

0.2 proper choice of P_1,P_2.
0.2 slope.
0.2 significant figures.
0.2 linear function.
0.1 unit of T_c.
0.6 right value of T_c
(0.6:160~180)
(0.3:150~160,180~190)
0 otherwise.

Note: other reasonable method that yields correct result is acceptable.

Alternative:</td>
</tr>
</table>

<table>
<tr>
<td></td>
<td>
</td>
<td>1.0 at most.
0.3 extent straight line. to intercept with x axis.
0.1 unit of T_c.
0.6 right value of T_c.</td>
</tr>
</table>

Experiment D

D.1	Assume that the length of the aluminum rod is L and the wave velocity is u. Under the free boundary condition, derive the equation for the frequencies f_n of the standing (resonant) waves along the long rod. Then derive the equation for the wave velocity u from f_n. Consider the aluminum rod as a one dimensional long string with free Boundary condition, then the standing wave condition is $$L = n\frac{\lambda}{2}, n = 1,2,3,\ldots \qquad (1)$$ According to $$\lambda f = u \qquad (2)$$ The standing wave frequencies are $$f_n = n\frac{u}{2L}, n = 1,2,3,\ldots \qquad (3)$$ Continually changing the vibration frequency, we can find out a series of standing wave frequencies f_n and calculate the average distance between two peaks $\overline{\Delta f}$, we have $$u = 2L\overline{\Delta f} \qquad (4)$$	Total:**0.6** **0.2** eqn.(1). **0.1** eqn.(2). **0.1** eqn.(3). **0.2** eqn.(4). Note: express u in terms of f_n instead of Δf is also acceptable.

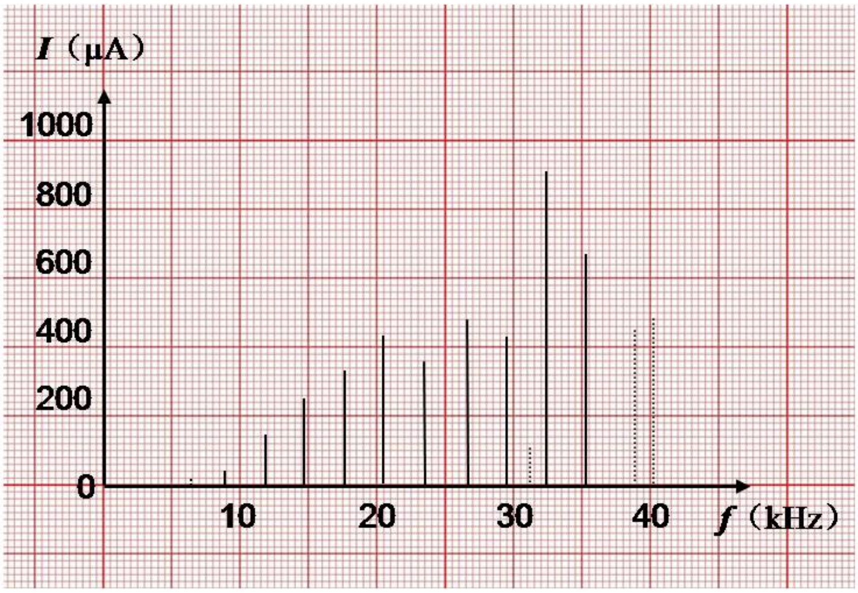
D.2

Use the steel tape measure to read the length L of the aluminum rod. Please repeat the measurement several times and calculate the mean and the standard error.

While changing the frequency of the sound waves produced by the transducer, record the peak values monitored by the sensor. Draw a spectrum containing all measured resonant peaks, similar to that shown in Figure 12.

	1	2	3	4	5	6	Avg
L(cm)	49.93	49.98	50.00	50.02	50.05	50.03	50.00±0.05

f(kHz)	6.37	8.81	11.81	14.70	17.54	20.45	23.44
I(μA)	16.5	42.6	144.0	249.5	336.9	247.7	358.4
f(kHz)	26.40	29.47	31.14	32.35	35.22	38.76	40.13
I(μA)	296.2	429.9	109.2	907	671	446.8	479



Total:**1.6**

0.1 multi measurement.
0.1 value of L.
0.1 error of L.

0.1 units.
0.1 significant figures.
0.1 ≥10 data points.
0.4 standing wave peaks resulting from the transverse waves
(**0.4**:≥6 peaks)
(**0.2**:3~5peaks)
0 otherwise.

0.3 frequency resolution 0.01kHz.

0.1 at least one miscellaneous peak.

0.2 spectrum containing all measured peaks.

D.3

Identify the resonant peaks likely resulting from the transverse waves. Calculate the transverse wave velocity accordingly and carry out the error analysis.

Attention: there might be irrelevant peaks caused by imperfection of the experimental setup, e.g., imperfect free boundary condition. You need to make a judgement and ignore the irrelevant peaks during your analysis.

i	f_i(kHz)	$F_i = f_{i+5} - f_i$ (kHz)		ΔF_i (kHz)	
1	8.81	$F_1 = f_6 - f_1$	14.63	ΔF_1	0.08
2	11.81	$F_2 = f_7 - f_2$	14.59	ΔF_2	0.12
3	14.70	$F_3 = f_8 - f_3$	14.77	ΔF_3	0.06
4	17.54	$F_4 = f_9 - f_4$	14.81	ΔF_4	0.10
5	20.45	$F_5 = f_{10} - f_5$	14.77	ΔF_5	0.06
6	23.44	$\overline{F}$	14.71	$\sigma_{\overline{\Delta F}}$	0.04
7	26.40				
8	29.47				
9	32.35				
10	35.22				

$$\overline{\Delta f} = \frac{\overline{F}}{5} = 2.94 \text{ kHz}$$

$$\sigma_{\overline{\Delta f}} = \frac{\sigma_{\overline{\Delta F}}}{5} = \frac{1}{5}\sqrt{\frac{\sum(\Delta F_i)^2}{n(n-1)}} = 0.01 \text{ kHz}$$

$$u = 2L\overline{\Delta f} = 2.94 \text{ km/s}$$

$$\frac{\Delta u}{u} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta f}{f}\right)^2} = 0.0035$$

$$\Delta u = 0.01 \text{ km/s}$$

$$u = (2.94 \pm 0.01) \text{ km/s}$$

Total: **1.4**

0.3 successive difference method.

Note: other reasonable method that yields correct result is acceptable.

0.1 data table.

0.1 significant figures.

0.1 unit.

0.6 right value of transverse wave velocity
(**0.6** 2.80~3.10 km/s)
(**0.2** 2.65~2.80 km/s, 3.10~3.25 km/s)
0 otherwise.

0.2 right value of Δu
(**0.2**:0.01~0.15 km/s)
(**0.1**:0.15~0.30 km/s)
0 otherwise.

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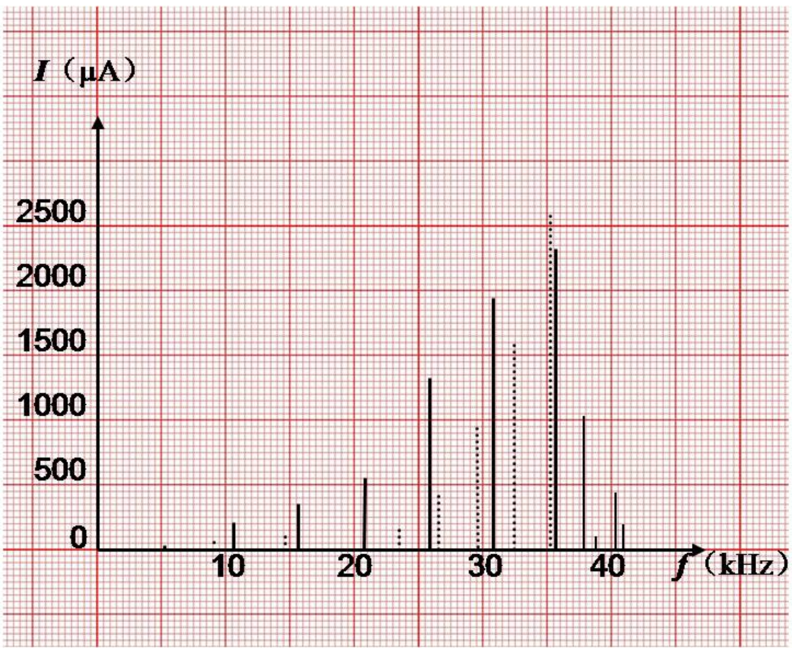
D.4	While changing the frequency of the sound waves produced by the transducer, record the peak values monitored by the sensor. Draw a spectrum containing all measured resonant peaks, similar to that shown in Figure 12.	Total:**1.5**

f(kHz)	5.18	8.90	10.50	14.53	15.53	20.63	23.39	24.68	25.67
I(μA)	15.5	52.7	216.2	103.6	353.3	555	156.4	45.8	1328
f(kHz)	26.44	29.40	30.64	32.33	35.20	35.55	37.81	38.74	40.26
I(μA)	414.5	848	1940	1593	2589	2331	1043	118.8	450
f(kHz)	40.79	41.53							
I(μA)	194.5	32.7							



0.1 unit.
0.1 significant figures.
0.1 >10 data points.
0.4 standing wave peaks resulting from the longitudinal waves
(**0.4**:≥6 peaks)
(**0.2**:3~5peaks)
0 otherwise.

0.2 standing wave peaks resulting from the transverse waves
(**0.2**:≥4 peaks)
(**0.1**:2~3peaks)
0 otherwise.

0.3 frequency resolution 0.01kHz.

0.1 at least one miscellaneous peak.

0.2 spectrum containing all measured peaks.

D.5

Compare with the result in D.2, identify the resonant peaks caused by the transverse waves. Select the resonant peaks resulting from the longitudinal waves and calculate the longitudinal wave velocity accordingly. Carry out the error analysis.

f_i(kHz)	$F_i = f_{i+3} - f_i$ (kHz)		ΔF_i (kHz)	
10 50	$F_1 = f_4 - f_1$	15.17	ΔF_1	0.10
15.53	$F_2 = f_5 - f_2$	15.11	ΔF_2	0.04
20.63	$F_3 = f_6 - f_3$	14.92	ΔF_3	0.15
25.67	$\overline{F}$	15.07	$\sigma_{\overline{\Delta F}}$	0.08
30.64				
35.55				

$$\overline{\Delta f} = \frac{\overline{F}}{3} = 5.02 \text{ kHz}$$

$$\sigma_{\overline{\Delta f}} = \frac{\sigma_{\overline{\Delta F}}}{3} = \frac{1}{3}\sqrt{\frac{\sum(\Delta F_i)^2}{n(n-1)}} = 0.03 \text{ kHz}$$

$$u = 2L\overline{\Delta f} = 5.02 \text{km/s}$$

$$\frac{\Delta u}{u} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta f}{f}\right)^2} = 0.006$$

$$\Delta u = 0.03 \text{ km/s}$$

Thus the longitudinal wave velocity is given by

$$u = 5.02 \pm 0.03 \text{ km/s}$$

Total:**1.4**

0.3 successive difference method.

Note: other reasonable method that yields correct result is acceptable.

0.1 data table.

0.1 significant figures.

0.1 units.

0.6 right value of longitudinal wave velocity
(**0.6**:4.70~5.20 km/s)
(**0.3**:4.50~4.70 km/s,5.20~5.40 km/s)
0 otherwise.

0.2 right value of Δu
(**0.2**:0.01~0.20 km/s)
(**0.1**:0.20~0.40 km/s)
0 otherwise.

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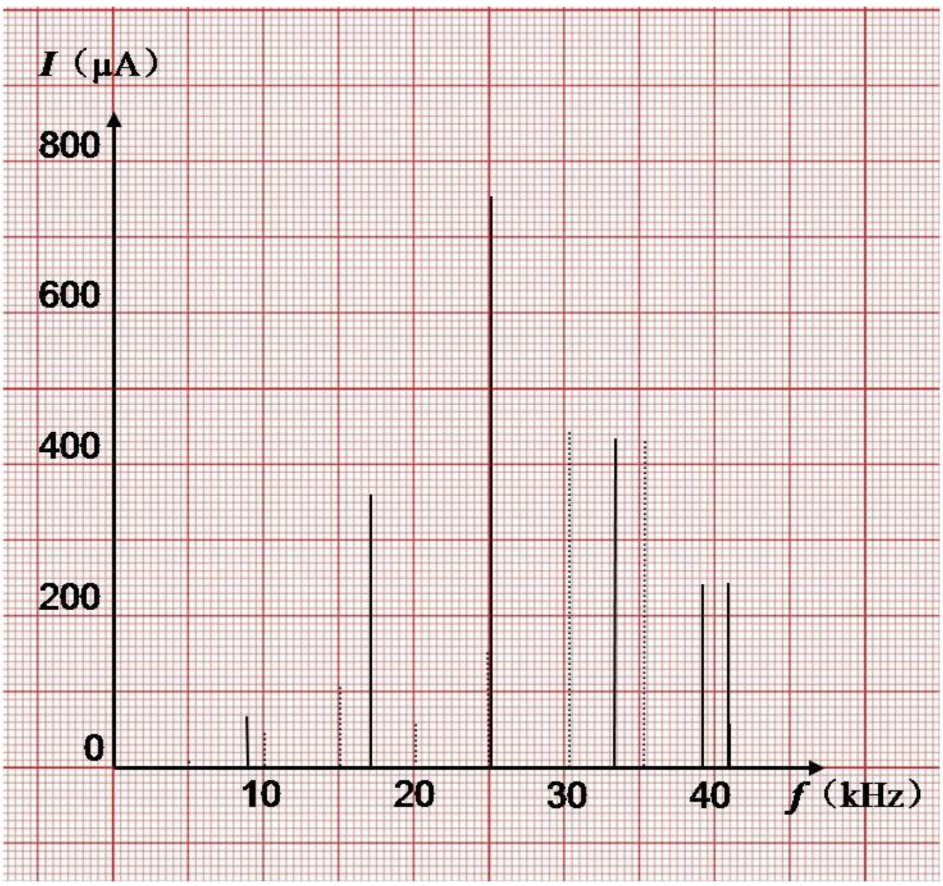
Experiment E

<table>
<tr><td>E.1</td><td>While changing the frequency of the sound waves produced by the transducer, record the peak values monitored by the sensor. Draw a spectrum containing all measured resonant peaks, similar to that shown in Figure 12.

f(kHz)	5.06	8.01	8.85	10.09	15.05	15.14	17.15	20.11
I(μA)	9.1	6.1	66.0	43.8	34.9	105.2	358.9	57.2
f(kHz)	24.95	25.17	30.27	33.37	35.27	39.23	40.82	41.05
I(μA)	150.9	751	441	432.7	430.1	241.9	242.4	57.7


</td><td>Total:**1.2**

0.1 unit.
0.1 significant figures.
0.1 >10 data points.
0.2 resonant peaks of the longitudinal waves corresponding to the cut
(**0.2**:≥4 peaks)
(**0.1**:2~3peaks)
0 otherwise.

0.1 at least 2 resonant peaks of the transverse waves.

0.3 frequency resolution 0.01kHz.

0.1 at least one miscellaneous peak.

0.2 spectrum containing all measured peaks.</td></tr>
</table>

<table>
<tr><td>E.2</td><td>In the measured spectrum, identify the resonant peaks corresponding to the existence of the deep cut. Estimate the distance from the spot of the cut to the end of the rod that is in contact with the PZT plates.

<table>
<tr><td>f(kHz)</td><td>8.85</td><td>17.15</td><td>25.17</td><td>33.37</td><td>40.82</td></tr>
<tr><td>I(μA)</td><td>66.0</td><td>358.9</td><td>751</td><td>432.7</td><td>242.4</td></tr>
<tr><td>$f_{i+2}-f_i$(kHz)</td><td colspan="5">16.32   16.22   15.65</td></tr>
<tr><td>$\overline{\Delta f}$ (kHz)</td><td colspan="5">8.03</td></tr>
</table>

$$\overline{\Delta f} = \frac{1}{2}\overline{F} = 8.11 \text{ kHz}$$

$$L_{\text{flaw}} = \frac{u}{2\overline{\Delta f}} = 0.307 \text{ m}$$
</td><td>Total:**0.8**

0.1 significant figures.
0.1 units.
0.6 right value of distance
(**0.6** 0.28~0.32m)
(**0.3** 0.26~0.28m, 0.32~0.34m)
0 otherwise.</td></tr>
</table>