## Theory Question 2: Watching a Rod in Motion

Enter all your answers into the Answer Script.


Physical situation A pinhole camera, with the pinhole at $x=0$ and at distance $D$ from the $x$ axis, takes pictures of a rod, by opening the pinhole for a very short time. There are equidistant marks along the $x$ axis by which the apparent length of the rod, as it is seen on the picture, can be determined from the pictures taken by the pinhole camera. On a picture of the rod at rest, its length is $L$. However, the rod is not at rest, but is moving with constant velocity $v$ along the $x$ axis.

Basic relations A picture taken by the pinhole camera shows a tiny segment of the rod at position $\tilde{x}$.
2.1 (0.6) What is the actual position $x$ of this segment at the time when the picture is taken? State your answer in terms of $\tilde{x}, D, L, v$, and the speed of light $c=3.00 \times 10^{8} \mathrm{~ms}^{-1}$. Employ the quantities

$$
\beta=\frac{v}{c} \text { and } \gamma=\frac{1}{\sqrt{1-\beta^{2}}}
$$

if they help to simplify your result.
2.2 (0.9) Find also the corresponding inverse relation, that is: express $\tilde{x}$ in terms of $x$, $D, L, v$, and $c$.
Note: The actual position is the position in the frame in which the camera is at rest
Apparent length of the rod The pinhole camera takes a picture at the instant when the actual position of the center of the rod is at some point $x_{0}$.
2.3 (1.5) In terms of the given variables, determine the apparent length of the rod on this picture.
2.4 (1.5) Check one of the boxes in the Answer Script to indicate how the apparent length changes with time.

Symmetric picture One pinhole-camera picture shows both ends of the rod at the same distance from the pinhole.
2.5 (0.8) Determine the apparent length of the rod on this picture.
2.6 (1.0) What is the actual position of the middle of the rod at the time when this picture is taken?
2.7 (1.2) Where does the picture show the image of the middle of the rod?

Very early and very late pictures The pinhole camera took one picture very early, when the rod was very far away and approaching, and takes another picture very late, when the rod is very far away and receding. On one of the pictures the apparent length is 1.00 m , on the other picture it is 3.00 m .
2.8 (0.5) Check the box in the Answer Script to indicate which length is seen on which picture.
2.9 (1.0) Determine the velocity $v$.
2.10 (0.6) Determine the length $L$ of the rod at rest.
2.11 (0.4) Infer the apparent length on the symmetric picture.

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|  |  | $\mathbf{2}$ |

## Answer Script

## Basic Relations

2.1 $x$ value for given $\tilde{x}$ value:

$$
x=
$$

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0.6
0.9
$\tilde{x}=$

Apparent length of the rod
2.3 The apparent length is

$$
\widetilde{L}\left(x_{0}\right)=
$$

2.4 Check one: The apparent length
1.5
$\square$ increases first, reaches a maximum value, then decreases.
$\square$ decreases first, reaches a minimum value, then increases.
$\square$ decreases all the time.
$\square$ increases all the time.

| Country Code | Student Code | Question Number |
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## Symmetric picture

2.5 The apparent length is
$\widetilde{L}=$
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0.8
1.0

$$
x_{0}=
$$

2.7 The picture shows the middle of the rod at a distance
$l=$
from the image of the front end of the rod.

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|  |  | $\mathbf{2}$ |

2.8 Check one:

## Very early and very late pictures

$\square$ The apparent length is 1 m on the early picture and 3 m on the late picture.
$\square$ The apparent length is 3 m on the early picture and 1 m on the late picture.
2.9 The velocity is
$v=$
2.10 The rod has length
$L=$
at rest.
2.11 The apparent length on the symmetric picture is

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Only
0.5

U
1.0
0.6
0.4
$\widetilde{L}=$

