Theory Question 3

This question consists of five independent parts. Each of them asks for an estimate of an order of magnitude only, not for a precise answer. Enter all your answers into the **Answer Script**.

Digital Camera Consider a digital camera with a square CCD chip with linear dimension L = 35 mm having N_p = 5 Mpix (1 Mpix = 10^6 pixels). The lens of this camera has a focal length of $f = 38$ mm. The well known sequence of numbers (2, 2.8, 4, 5.6, 8, 11, 16, 22) that appear on the lens refer to the so called F-number, which is denoted by $F\#$ and defined as the ratio of the focal length and the diameter D of the aperture of the lens, $F\# = f / D$.

3.1 (1.0) Find the best possible spatial resolution $\Delta x_{\min}$, at the chip, of the camera as limited by the lens. Express your result in terms of the wavelength λ and the F-number $F\#$ and give the numerical value for $\lambda = 500$ nm.

3.2 (0.5) Find the necessary number N of Mpix that the CCD chip should possess in order to match this optimal resolution.

3.3 (0.5) Sometimes, photographers try to use a camera at the smallest practical aperture. Suppose that we now have a camera of $N_0 = 16$ Mpix, with the chip size and focal length as given above. Which value is to be chosen for $F\#$ such that the image quality is not limited by the optics?

3.4 (0.5) Knowing that the human eye has an approximate angular resolution of $\phi = 2$ arcsec and that a typical photo printer will print a minimum of 300 dpi (dots per inch), at what minimal distance z should you hold the printed page from your eyes so that you do not see the individual dots?

<u>Data</u> 1 inch = 25.4 mm
1 arcsec = 2.91×10^{-4} rad

Hard-boiled egg An egg, taken directly from the fridge at temperature $T_0 = 4°C$, is dropped into a pot with water that is kept boiling at temperature T_1.

3.5 (0.5) How large is the amount of energy U that is needed to get the egg coagulated?

3.6 (0.5) How large is the heat flow J that is flowing into the egg?

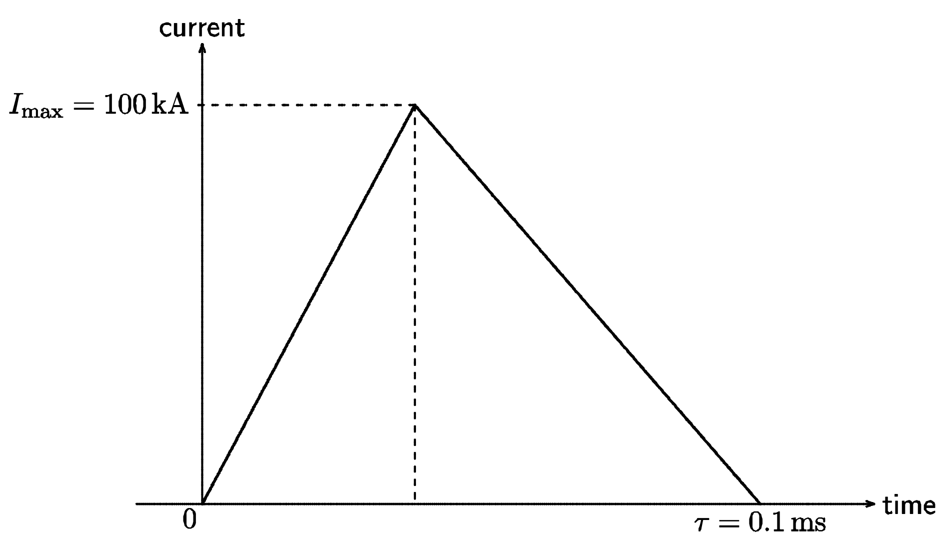
3.7 (0.5) How large is the heat power P transferred to the egg?

3.8 (0.5) For how long do you need to cook the egg so that it is hard-boiled?

Hint You may use the simplified form of Fourier's Law $J = \kappa \Delta T / \Delta r$, where ΔT is the temperature difference associated with Δr, the typical length scale of the problem. The heat flow J is in units of $\mathrm{W\,m^{-2}}$.

Data Mass density of the egg: $\mu = 10^3\ \mathrm{kg\,m^{-3}}$
Specific heat capacity of the egg: $C = 4.2\ \mathrm{J\,K^{-1}\,g^{-1}}$
Radius of the egg: $R = 2.5\ \mathrm{cm}$
Coagulation temperature of albumen (egg protein): $T_c = 65°C$
Heat transport coefficient: $\kappa = 0.64\ \mathrm{W\,K^{-1}\,m^{-1}}$ (assumed to be the same for liquid and solid albumen)

Lightning An oversimplified model of lightning is presented. Lightning is caused by the build-up of electrostatic charge in clouds. As a consequence, the bottom of the cloud usually gets positively charged and the top gets negatively charged, and the ground below the cloud gets negatively charged. When the corresponding electric field exceeds the breakdown strength value of air, a disruptive discharge occurs: this is lightning.



Idealized current pulse flowing between the cloud and the ground during lightning.

Answer the following questions with the aid of this simplified curve for the current as a function of time and these data:

Distance between the bottom of the cloud and the ground: h = 1 km;
Breakdown electric field of humid air: $E_0 = 300\,\text{kV m}^{-1}$;
Total number of lightning striking Earth per year: 32×10^6;
Total human population: 6.5×10^9 people.

3.9 (0.5) What is the total charge Q released by lightning?

3.10 (0.5) What is the average current I flowing between the bottom of the cloud and the ground during lightning?

3.11 (1.0) Imagine that the energy of all storms of one year is collected and equally shared among all people. For how long could you continuously light up a 100 W light bulb for your share?

Capillary Vessels Let us regard blood as an incompressible viscous fluid with mass density μ similar to that of water and dynamic viscosity η = 4.5 g m^{-1} s^{-1}. We model blood vessels as circular straight pipes with radius r and length L and describe the blood flow by Poiseuille's law,

$$\Delta p = RD\,,$$

the Fluid Dynamics analog of Ohm's law in Electricity. Here Δp is the pressure difference between the entrance and the exit of the blood vessel, $D = S\upsilon$ is the volume flow through the cross-sectional area S of the blood vessel and υ is the blood velocity. The hydraulic resistance R is given by

$$R = \frac{8\eta L}{\pi r^4}\,.$$

For the systemic blood circulation (the one flowing from the left ventricle to the right auricle of the heart), the blood flow is $D \approx 100$ cm^{3}s^{-1} for a man at rest. Answer the following questions under the assumption that all capillary vessels are connected in parallel and that each of them has radius r = 4 µm and length L = 1 mm and operates under a pressure difference $\Delta p = 1$ kPa.

3.12 (1.0) How many capillary vessels are in the human body?

3.13 (0.5) How large is the velocity υ with which blood is flowing through a capillary vessel?

Skyscraper At the bottom of a 1000 m high skyscraper, the outside temperature is $T_{\text{bot}} = 30°\text{C}$. The objective is to estimate the outside temperature T_{top} at the top. Consider a thin slab of air (ideal nitrogen gas with adiabatic coefficient $\gamma = 7/5$) rising slowly to height z where the pressure is lower, and assume that this slab expands adiabatically so that its temperature drops to the temperature of the surrounding air.

3.14 (0.5) How is the fractional change in temperature dT/T related to dp/p, the fractional change in pressure?

3.15 (0.5) Express the pressure difference dp in terms of dz, the change in height.

3.16 (1.0) What is the resulting temperature at the top of the building?

<u>Data</u> Boltzmann constant: $k = 1.38 \times 10^{-23}\ \text{J K}^{-1}$
Mass of a nitrogen molecule: $m = 4.65 \times 10^{-26}\ \text{kg}$
Gravitational acceleration: $g = 9.80\ \text{m s}^{-2}$

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Answer Script

Digital Camera

	For Examiners Use Only
3.1 The best spatial resolution is (formula:) $\Delta x_{\min} =$	**0.7**
which gives (numerical:) $\Delta x_{\min} =$	**0.3**
for $\lambda = 500\,\text{nm}$.	
3.2 The number of Mpix is $N =$	**0.5**
3.3 The best F-number value is $F\# =$	**0.5**
3.4 The minimal distance is $z =$	**0.5**

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Hard-boiled egg

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3.5 The required energy is

$U =$

0.5

3.6 The heat flow is

$J =$

0.5

3.7 The heat power transferred is

$P =$

0.5

3.8 The time needed to hard-boil the egg is

$\tau =$

0.5

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Lightning

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3.9 The total charge is $Q =$	**0.5**
3.10 The average current is $I =$	**0.5**
3.11 The light bulb would burn for the duration $t =$	**1.0**

Capillary Vessels

3.12 There are $N =$ capillary vessels in a human body.	**1.0**
3.13 The blood flows with velocity $\upsilon =$	**0.5**

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Skyscraper

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3.14 The fractional change in temperature is

$$\frac{dT}{T} =$$

0.5

3.15 The pressure difference is

$$dp =$$

0.5

3.16 The temperature at the top is

$$T_{\text{top}} =$$

1.0