2. Mechanical Blackbox: a cylinder with a ball inside

A small massive particle (ball) of mass m is fixed at distance z below the top of a long hollow cylinder of mass M. A series of holes are drilled perpendicularly to the central axis of the cylinder. These holes are for pivoting so that the cylinder will hang in a vertical plane.

Students are required to perform necessary nondestructive measurements to determine the numerical values of the following with their error estimates:

i. position of centre of mass of cylinder with ball inside.

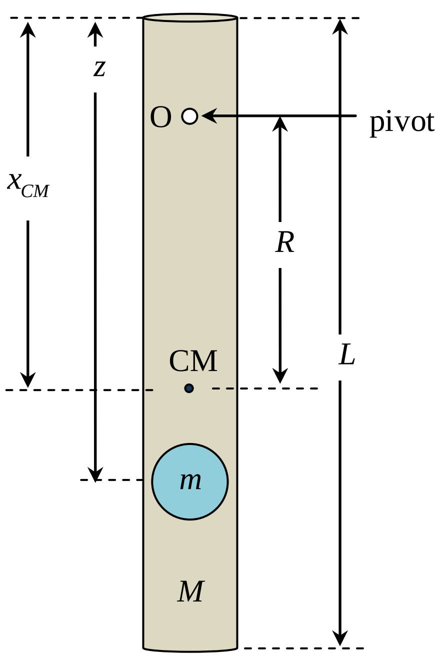
Also provide a schematic drawing of the experimental set-up for measuring the centre of mass. **[1.0 points]**

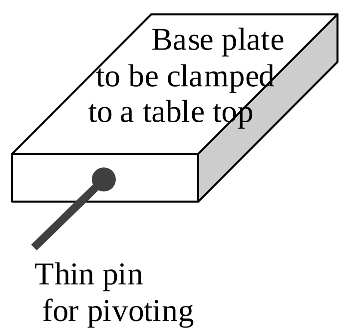
ii. distance z **[3.5 points]**

iii. ratio $\frac{M}{m}$. **[3.5 points]**

iv. the acceleration due to gravity, g. **[2.0 points]**

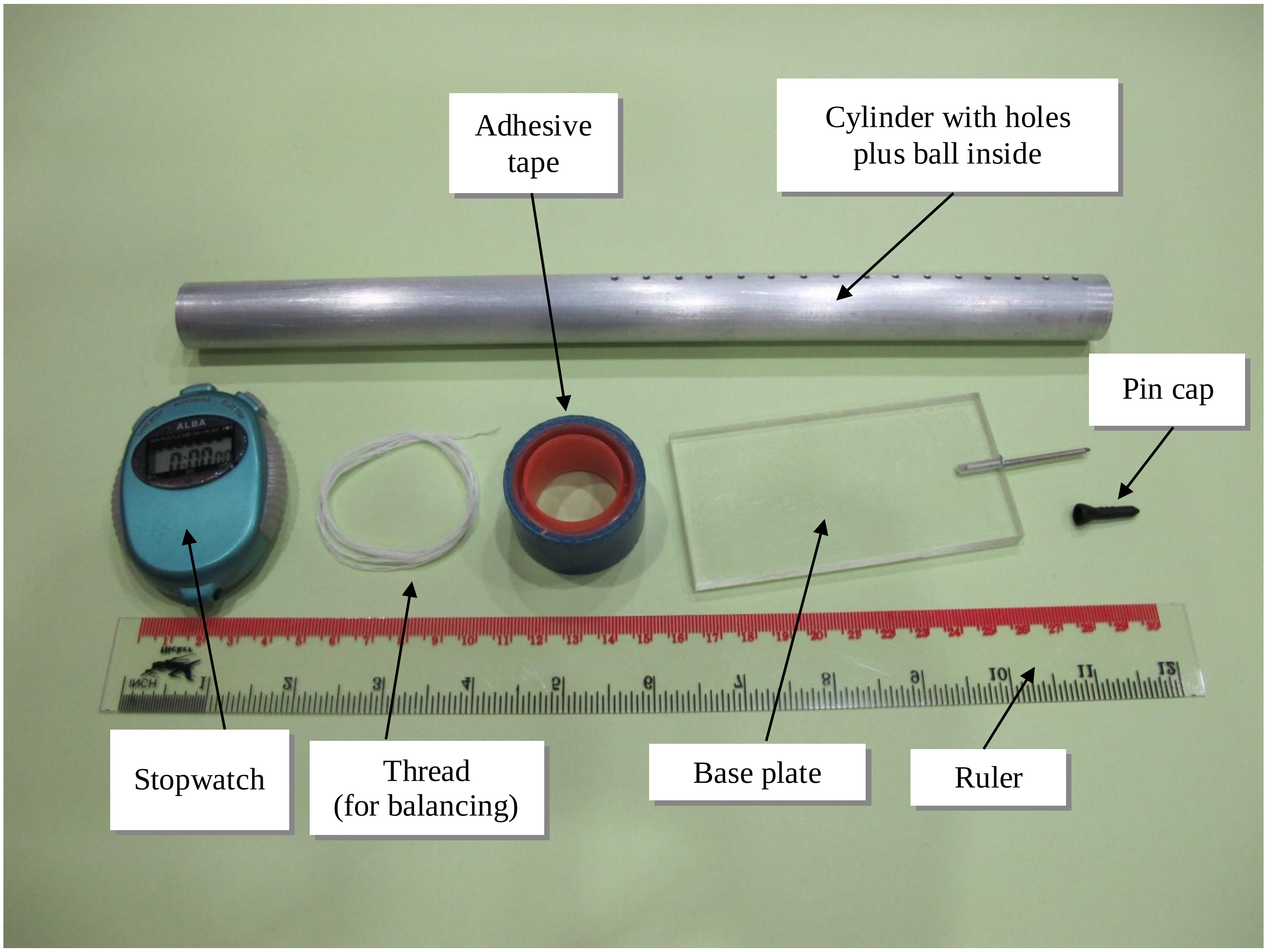
Equipment: a cylinder with holes plus a ball inside, a base plate with a thin pin, a pin cap, a ruler, a stopwatch, thread, a pencil and adhesive tape.





x_{CM} is the distance from the top of the cylinder to the centre of mass.

R is the distance from the pivoting point to the centre of mass.



Caution: The thin pin is sharp. When it is not in use, it should be protected with a pin cap for safety.

Useful information:

1. For such a physical pendulum, $\left[(M+m)R^2+I_{CM}\right]\dfrac{d^2\theta}{dt^2} \approx -g(M+m)R\theta$, where I_{CM} is the moment of inertia of the cylinder with a ball about the centre of mass and θ is the angular displacement.
2. For a long hollow cylinder of length L and mass M, the moment of inertia about the centre of mass with the rotational axis perpendicular to the cylinder can be approximated by $\dfrac{1}{3}M\left(\dfrac{L}{2}\right)^2$.
3. The parallel axis theorem: $I = I_{\text{centre of mass}} + \mathfrak{M}x^2$, where x is the distance from the rotation point to the centre of mass, and $\mathfrak{M}$ is the total mass of the object.
4. The ball can be treated as a point mass and it is located on the central axis of the cylinder.
5. Assume that the cylinder is uniform and the mass of the end-caps is negligible.