3. To Commemorate the Centenary of Rutherford's Atomic Nucleus: the Scattering of an Ion by a Neutral Atom

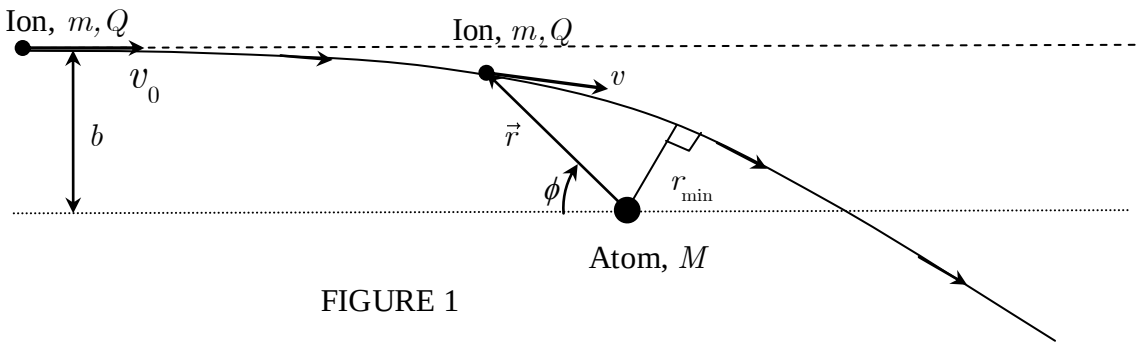

FIGURE 1

An ion of mass m, charge Q, is moving with an initial non-relativistic speed v_0 from a great distance towards the vicinity of a neutral atom of mass $M >> m$ and of electrical polarisability α. The impact parameter is b as shown in Figure 1.

The atom is instantaneously polarised by the electric field $\vec{E}$ of the in-coming (approaching) ion. The resulting electric dipole moment of the atom is $\vec{p} = \alpha \vec{E}$. Ignore any radiative losses in this problem.

3.1 Calculate the electric field intensity $\vec{E}_p$ at a distance r from an ideal electric dipole $\vec{p}$ at the origin O along the direction of $\vec{p}$ in Figure 2. **[1.2 points]**

$p = 2aq, \quad r \gg a$

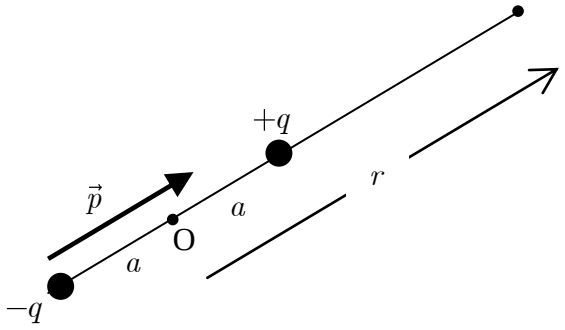

FIGURE 2

3.2 Find the expression for the force $\vec{f}$ acting on the ion due to the polarised atom. Show that this force is attractive regardless of the sign of the charge of the ion.

[3.0 points]

3.3 What is the electric potential energy of the ion-atom interaction in terms of α, Q and r?

[0.9 points]

3.4 Find the expression for $r_{\min}$, the distance of the closest approach, as shown in Figure 1.

[2.4 points]

3.5 If the impact parameter b is less than a critical value b_0, the ion will descend along a spiral to the atom. In such a case, the ion will be neutralized, and the atom is, in turn, charged. This process is known as the "charge exchange" interaction. What is the cross sectional area $A = \pi b_0^2$ of this "charge exchange" collision of the atom as seen by the ion? **[2.5 points]**