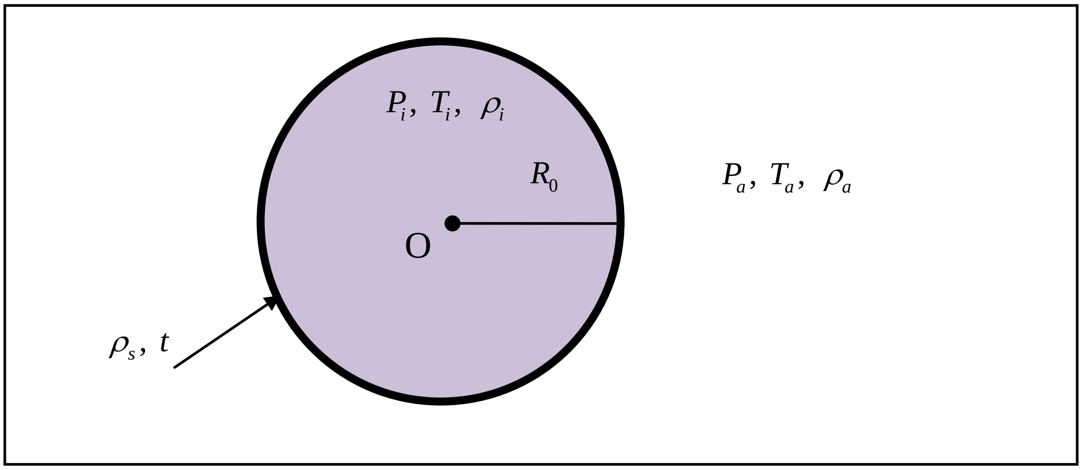
2. SOLUTION

2.1. The bubble is surrounded by air.



Cutting the sphere in half and using the projected area to balance the forces give

$$P_i \pi R_0^2 = P_a \pi R_0^2 + 2\left(2\pi R_0 \gamma\right)$$
$$P_i = P_a + \frac{4\gamma}{R_0} \qquad \ldots (1)$$

The pressure and density are related by the ideal gas law:

$PV = nRT$ or $P = \dfrac{\rho RT}{M}$, where M = the molar mass of air. … (2)

Apply the ideal gas law to the air inside and outside the bubble, we get

$$\rho_i T_i = P_i \frac{M}{R}$$
$$\rho_a T_a = P_a \frac{M}{R},$$

$$\frac{\rho_i T_i}{\rho_a T_a} = \frac{P_i}{P_a} = \left[1 + \frac{4\gamma}{R_0 P_a}\right] \qquad \ldots (3)$$

2.2. Using $\gamma = 0.025\,\text{Nm}^{-1}$, $R_0 = 1.0$ cm and $P_a = 1.013\times10^5\,\text{Nm}^{-2}$, the numerical value of the ratio is

$$\frac{\rho_i T_i}{\rho_a T_a} = 1 + \frac{4\gamma}{R_0 P_a} = 1 + 0.0001 \qquad \ldots (4)$$

(The effect of the surface tension is very small.)

2.3. Let W = total weight of the bubble, F = buoyant force due to air around the bubble

$$\begin{aligned} W &= (\text{mass of film+mass of air})\,g \\ &= \left(4\pi R_0^2\,\rho_s t + \frac{4}{3}\pi R_0^3 \rho_i\right) g \qquad \ldots (5) \\ &= 4\pi R_0^2\,\rho_s t g + \frac{4}{3}\pi R_0^3 \frac{\rho_a T_a}{T_i}\left[1 + \frac{4\gamma}{R_0 P_a}\right] g \end{aligned}$$

The buoyant force due to air around the bubble is

$$B = \frac{4}{3}\pi R_0^3 \rho_a g \qquad \ldots (6)$$

If the bubble floats in still air,

$$B \ge W$$

$$\frac{4}{3}\pi R_0^3 \rho_a g \ge 4\pi R_0^2\,\rho_s t g + \frac{4}{3}\pi R_0^3 \frac{\rho_a T_a}{T_i}\left[1 + \frac{4\gamma}{R_0 P_a}\right] g \qquad \ldots (7)$$

Rearranging to give

$$\begin{aligned} T_i &\ge \frac{R_0 \rho_a T_a}{R_0 \rho_a - 3\rho_s t}\left[1 + \frac{4\gamma}{R_0 P_a}\right] \qquad \ldots (8) \\ &\ge 307.1\ \text{K} \end{aligned}$$

The air inside must be about $7.1°\text{C}$ warmer.

2.4. Ignore the radius change → Radius remains $R_0 = 1.0\ \text{cm}$

(The radius actually decreases by 0.8% when the temperature decreases from 307.1 K to 300 K. The film itself also becomes slightly thicker.)

The drag force from Stokes' Law is $F = 6\pi\eta R_0 u$ … (9)

If the bubble floats in the updraught,

$$F \geq W - B$$

$$6\pi\eta R_0 u \geq \left(4\pi R_0^2\,\rho_s t + \frac{4}{3}\pi R_0^3 \rho_i\right) g - \frac{4}{3}\pi R_0^3 \rho_a g \quad \text{… (10)}$$

When the bubble is in thermal equilibrium $T_i = T_a$.

$$6\pi\eta R_0 u \geq \left(4\pi R_0^2\,\rho_s t + \frac{4}{3}\pi R_0^3 \rho_a \left[1 + \frac{4\gamma}{R_0 P_a}\right]\right) g - \frac{4}{3}\pi R_0^3 \rho_a g$$

Rearranging to give

$$u \geq \frac{4R_0\,\rho_s t g}{6\eta} + \frac{\frac{4}{3} R_0^2 \rho_a g \left(\frac{4\gamma}{R_0 P_a}\right)}{6\eta} \quad \text{… (11)}$$

2.5. The numerical value is $u \geq 0.36\ \text{m/s}$.

The 2nd term is about 3 orders of magnitude lower than the 1st term.

From now on, ignore the surface tension terms.

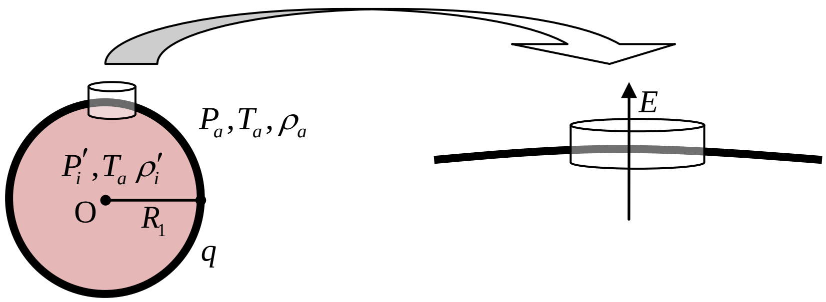
2.6. When the bubble is electrified, the electrical repulsion will cause the bubble to expand in size and thereby raise the buoyant force.

The force/area is (e-field on the surface × charge/area)

There are two alternatives to calculate the electric field ON the surface of the soap film.

A. From Gauss's Law

Consider a very thin pill box on the soap surface.



E = electric field on the film surface that results from all other parts of the soap film, excluding the surface inside the pill box itself.

E_q = total field just outside the pill box = $\dfrac{q}{4\pi\varepsilon_0 R_1^2} = \dfrac{\sigma}{\varepsilon_0}$

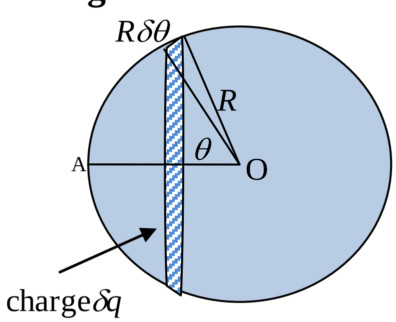
= E + electric field from surface charge σ

= $E + E_\sigma$

Using Gauss's Law on the pill box, we have $E_\sigma = \dfrac{\sigma}{2\varepsilon_0}$ perpendicular to the film as a result of symmetry.

Therefore, $E = E_q - E_\sigma = \dfrac{\sigma}{\varepsilon_0} - \dfrac{\sigma}{2\varepsilon_0} = \dfrac{\sigma}{2\varepsilon_0} = \dfrac{1}{2\varepsilon_0}\dfrac{q}{4\pi R_1^2}$ … (12)

B. From direct integration



$$\delta q = \left(\frac{q}{4\pi R^2}\right) 2\pi R \sin\theta . R\delta\theta$$

To find the magnitude of the electrical repulsion we must first find the electric field intensity E at a point <u>on</u> (not outside) the surface itself.

Field at A in the direction $\overrightarrow{OA}$ is

$$\delta E_A = \frac{1}{4\pi\varepsilon_0}\frac{\left(q/4\pi R_1^2\right)2\pi R_1^2\sin\theta\delta\theta}{\left(2R_1\sin\frac{\theta}{2}\right)^2}\sin\frac{\theta}{2} = \frac{\left(q/4\pi R_1^2\right)}{2\varepsilon_0}\cos\frac{\theta}{2}\delta\left(\frac{\theta}{2}\right)$$

$$E_A = \frac{\left(q/4\pi R_1^2\right)}{2\varepsilon_0}\int_{\theta=0}^{\theta=180^\circ}\cos\frac{\theta}{2}d\left(\frac{\theta}{2}\right) = \frac{\left(q/4\pi R_1^2\right)}{2\varepsilon_0} \quad \ldots (13)$$

The repulsive force per unit area of the surface of bubble is

$$\left(\frac{q}{4\pi R_1^2}\right)E = \frac{\left(q/4\pi R_1^2\right)^2}{2\varepsilon_0} \quad \ldots (14)$$

Let P_i' and ρ_i' be the new pressure and density when the bubble is electrified.

This electric repulsive force will augment the gaseous pressure P_i'.
P_i' is related to the original P_i through the gas law.

$$P_i'\frac{4}{3}\pi R_1^3 = P_i\frac{4}{3}\pi R_0^3$$

$$P_i' = \left(\frac{R_0}{R_1}\right)^3 P_i = \left(\frac{R_0}{R_1}\right)^3 P_a \quad \ldots (15)$$

<u>In the last equation, the surface tension term has been ignored.</u>

From balancing the forces on the half-sphere projected area, we have (again ignoring the surface tension term)

$$P_i' + \frac{\left(q/4\pi R_1^2\right)^2}{2\varepsilon_0} = P_a$$

$$P_a\left(\frac{R_0}{R_1}\right)^3 + \frac{\left(q/4\pi R_1^2\right)^2}{2\varepsilon_0} = P_a \quad \ldots (16)$$

Rearranging to get

$$\left(\frac{R_1}{R_0}\right)^4 - \left(\frac{R_1}{R_0}\right) - \frac{q^2}{32\pi^2\varepsilon_0 R_0^4 P_a} = 0 \qquad \dots (17)$$

Note that (17) yields $\frac{R_1}{R_0} = 1$ when $q = 0$, as expected.

2.7. Approximate solution for R_1 when $\frac{q^2}{32\pi^2\varepsilon_0 R_0^4 P_a} << 1$

Write $R_1 = R_0 + \Delta R,\ \Delta R << R_0$

Therefore, $\frac{R_1}{R_0} = 1 + \frac{\Delta R}{R_0},\ \left(\frac{R_1}{R_0}\right)^4 \approx 1 + 4\frac{\Delta R}{R_0}$ ... (18)

Eq. (17) gives:

$$\Delta R \approx \frac{q^2}{96\pi^2\varepsilon_0 R_0^3 P_a} \qquad \dots (19)$$

$$R_1 \approx R_0 + \frac{q^2}{96\pi^2\varepsilon_0 R_0^3 P_a} \approx R_0\left(1 + \frac{q^2}{96\pi^2\varepsilon_0 R_0^4 P_a}\right) \qquad \dots (20)$$

2.8. The bubble will float if

$$B \geq W$$

$$\frac{4}{3}\pi R_1^3 \rho_a g \geq 4\pi R_0^2\,\rho_s t g + \frac{4}{3}\pi R_0^3 \rho_i g \qquad \dots (21)$$

Initially, $T_i = T_a \Rightarrow \rho_i = \rho_a$ for $\gamma \to 0$ and $R_1 = R_0\left(1 + \frac{\Delta R}{R_0}\right)$

$$\frac{4}{3}\pi R_0^3\left(1+\frac{\Delta R}{R_0}\right)^3\rho_a g \geq 4\pi R_0^2\,\rho_s t g+\frac{4}{3}\pi R_0^3\rho_a g$$

$$\frac{4}{3}\pi\left(3\Delta R\right)\rho_a g \geq 4\pi R_0^2\,\rho_s t g$$

$$\frac{4}{3}\pi\frac{3q^2}{96\pi^2\varepsilon_0 R_0 P_a}\rho_a g \geq 4\pi R_0^2\,\rho_s t g \qquad \dots (22)$$

$$q^2 \geq \frac{96\pi^2 R_0^3\,\rho_s t\varepsilon_0 P_a}{\rho_a}$$

$$q \approx 256\times10^{-9}\ \text{C} \approx 256\,\text{nC}$$

Note that if the surface tension term is retained, we get

$$R_1 \approx \left(1+\frac{q^2/96\pi^2\varepsilon_0 R_0^4 P_a}{\left[1+\frac{2}{3}\left(\frac{4\gamma}{R_0 P_a}\right)\right]}\right)R_0$$