The 43rd International Physics Olympiad — Theoretical Competition

Tartu, Estonia — Tuesday, July 17th 2012

- The examination lasts for 5 hours. There are 3 problems worth a total of 30 points. Please note that the point values of the three theoretical problems are not equal.
- **You must not open the envelope with the problems before the sound signal indicating the beginning of competition (three short signals).**
- **You are not allowed to leave your working place without permission.** If you need any assistance (broken calculator, need to visit a restroom, etc), please raise the corresponding flag ("HELP" or "TOILET" with a long handle at your seat) above your seat box walls and keep it raised until an organizer arrives.
- **Your answers must be expressed in terms of those quantities, which are highlighted** in the problem text, and can contain also fundamental constants, if needed. So, if it is written that "the box height is a and the width - b" then a can be used in the answer, and b cannot be used (unless it is highlighted somewhere else, see below). Those quantities which are highlighted in the text of a subquestion can be used only in the answer to that subquestion; the quantities which are highlighted in the introductory text of the Problem (or a Part of a Problem), i.e. outside the scope of any subquestion, can be used for all the answers of that Problem (or of that Problem Part).
- Use only the front side of the sheets of paper.
- For each problem, there are **dedicated Solution Sheets** (see header for the number and pictogramme). Write your solutions onto the appropriate Solution Sheets. For each Problem, the Solution Sheets are numbered; use the sheets according to the enumeration. **Always mark which Problem Part and Question you are dealing with.** Copy the final answers into the appropriate boxes of the **Answer Sheets**. There are also **Draft** papers; use these for writing things which you don't want to be graded. If you have written something what you don't want to be graded onto the Solution Sheets (such as initial and incorrect solutions), cross these out.
- If you need more paper for a certain problem, please raise the flag "HELP" and tell an organizer the problem number; you are given two Solution sheets (you can do this more than once).
- **You should use as little text as possible:** try to explain your solution mainly with equations, numbers, symbols and diagrams.
- The first single sound signal tells you that there are 30 min of solving time left; the second double sound signal means that 5 min is left; the third triple sound signal marks the end of solving time. **After the third sound signal you must stop writing immediately.** Put all the papers into the envelope at your desk. **You are not allowed to take any sheet of paper out of the room.** If you have finished solving before the final sound signal, please raise your flag.

Problem T1. Focus on sketches *(13 points)*

Part A. Ballistics *(4.5 points)*

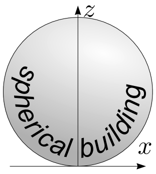
A ball, thrown with an initial speed v_0, moves in a homogeneous gravitational field in the x-z plane, where the x-axis is horizontal, and the z-axis is vertical and antiparallel to the free fall acceleration g. Neglect the effect of air drag.

i. (0.8 pts) By adjusting the launching angle for a ball thrown with a fixed initial speed v_0 from the origin, targets can be hit within the region given by

$$z \le z_0 - kx^2.$$

You can use this fact without proving it. Find the constants z_0 and k.

ii. (1.2 pts) The launching point can now be freely selected on the ground level $z = 0$, and the launching angle can be adjusted as needed. The aim is to hit the topmost point of a spherical building of radius R (see fig.) with the minimal initial speed v_0. Bouncing off the roof prior to hitting the target is not allowed. Sketch qualitatively the shape of the optimal trajectory of the ball (use the designated box on the answer sheet). Note that the marks are given only for the sketch.



iii. (2.5 pts) What is the minimal launching speed $v_{\min}$ needed to hit the topmost point of a spherical building of radius R?

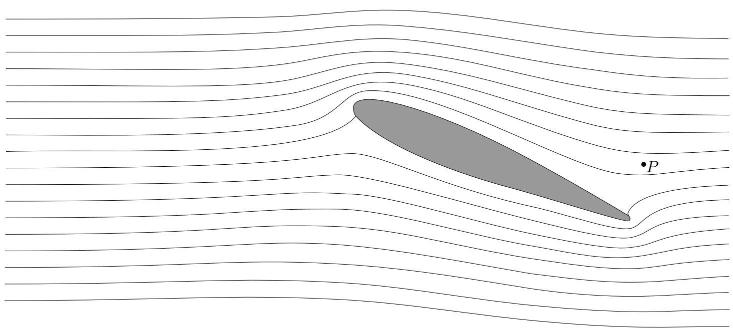
La Geode, Parc de la Villette, Paris. Photo: katchooo/flickr.com

Part B. Air flow around a wing *(4 points)*

For this part of the problem, the following information may be useful. For a flow of liquid or gas in a tube along a streamline, $p + \rho g h + \frac{1}{2}\rho v^2 = \text{const.}$, assuming that the velocity v is much less than the speed of sound. Here ρ is the density, h is the height, g is free fall acceleration and p is hydrostatic pressure. Streamlines are defined as the trajectories of fluid particles (assuming that the flow pattern is stationary). Note that the term $\frac{1}{2}\rho v^2$ is called the dynamic pressure.

In the fig. shown below, a cross-section of an aircraft wing is depicted together with streamlines of the air flow around the wing, as seen in the wing's reference frame. Assume that (a) the air flow is purely two-dimensional (i.e. that the velocity vectors of air lie in the plane of the figure); (b) the streamline pattern is independent of the aircraft speed; (c) there is no wind; (d) the dynamic pressure is much smaller than the atmospheric pressure, $p_0 = 1.0 \times 10^5\,\text{Pa}$.

You can use a ruler to take **measurements from the fig. on the answer sheet**.



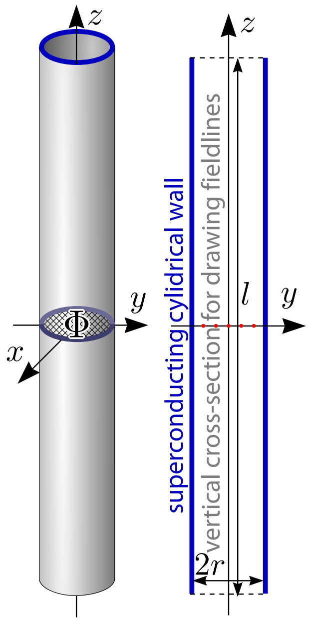
i. (0.8 pts) If the aircraft's ground speed is $v_0 = 100\,\text{m/s}$, what is the speed of the air, v_P, at the point P (marked in the fig.) with respect to the ground?

ii. (1.2 pts) In the case of high relative humidity, as the ground speed of the aircraft increases over a critical value v_{crit}, a stream of water droplets is created behind the wing. The droplets emerge at a certain point Q. Mark the point Q in the fig. on the answer sheet. Explain qualitatively (using formulae and as little text as possible) how you determined the position of Q.

iii. (2.0 pts) Estimate the critical speed v_{crit} using the following data: relative humidity of the air is $r = 90\%$, specific heat capacity of air at constant pressure $c_p = 1.00 \times 10^3\,\text{J/kg}\cdot\text{K}$, pressure of saturated water vapour: $p_{sa} = 2.31\,\text{kPa}$ at the temperature of the unperturbed air $T_a = 293\,\text{K}$ and $p_{sb} = 2.46\,\text{kPa}$ at $T_b = 294\,\text{K}$. Depending on your approximations, you may also need the specific heat capacity of air at constant volume $c_V = 0.717 \times 10^3\,\text{J/kg}\cdot\text{K}$. Note that the relative humidity is defined as the ratio of the vapour pressure to the saturated vapour pressure at the given temperature. Saturated vapour pressure is defined as the vapour pressure by which vapour is in equilibrium with the liquid.

Part C. Magnetic straws *(4.5 points)*

Consider a cylindrical tube made of a superconducting material. The length of the tube is l and the inner radius is r with $l \gg r$. The centre of the tube coincides with the origin, and its axis coincides with the z-axis.

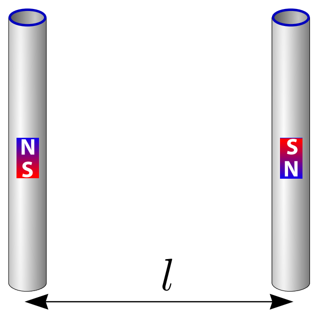


There is a magnetic flux Φ through the central cross-section of the tube, $z = 0$, $x^2 + y^2 < r^2$. A superconductor is a material which expels any magnetic field (the field is zero inside the material).

i. (0.8 pts) Sketch five such magnetic field lines, which pass through the five red dots marked on the axial cross-section of the tube, on the designated diagram on the answer sheet.

ii. (1.2 pts) Find the tension force T along the z-axis in the middle of the tube (i.e. the force by which two halves of the tube, $z > 0$ and $z < 0$, interact with each other).

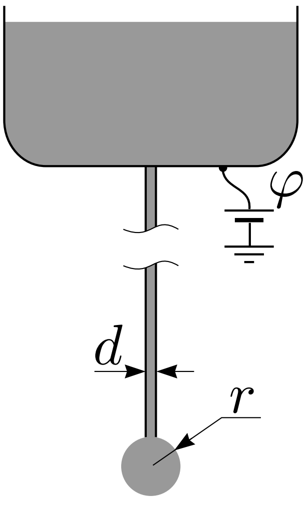
iii. (2.5 pts) Consider another tube, identical and parallel to the first one.



The second tube has the same magnetic field but in the opposite direction and its centre is placed at $y = l$, $x = z = 0$ (so that the tubes form opposite sides of a square). Determine the magnetic interaction force F between the two tubes.

Problem T2. Kelvin water dropper *(8 points)*

The following facts about the surface tension may turn out to be useful for this problem. For the molecules of a liquid, the positions at the liquid-air interface are less favourable as compared with the positions in the bulk of the liquid. This interface is described by the so-called surface energy, $U = \sigma S$, where S is the surface area of the interface and σ is the surface tension coefficient of the liquid. Moreover, two fragments of the liquid surface pull each other with a force $F = \sigma l$, where l is the length of a straight line separating the fragments.



A long metallic pipe with internal diameter d is pointing directly downwards. Water is slowly dripping from a nozzle at its lower end, see fig. Water can be considered to be electrically conducting; its surface tension is σ and its density is ρ. A droplet of radius r hangs below the nozzle. The radius grows slowly in time until the droplet separates from the nozzle due to the free fall acceleration g. Always assume that $d \ll r$.

Part A. Single pipe *(4 points)*

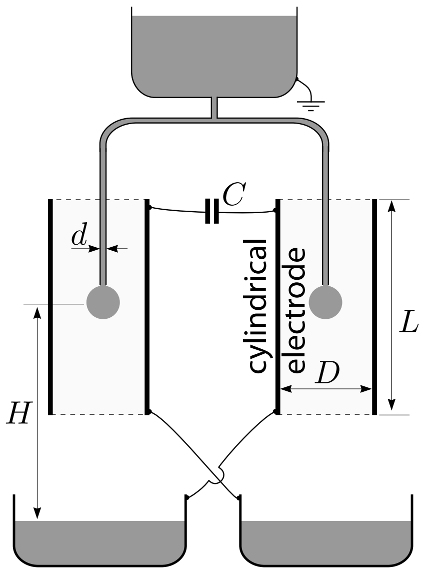
i. (1.2 pts) Find the radius $r_{\max}$ of a drop just before it separates from the nozzle.

ii. (1.2 pts) Relative to the far-away surroundings, the pipe's electrostatic potential is φ. Find the charge Q of a drop when its radius is r.

iii. (1.6 pts) Consider the situation in which r is kept constant and φ is slowly increased. The droplet becomes unstable and breaks into pieces if the hydrostatic pressure inside the droplet becomes smaller than the atmospheric pressure. Find the critical potential $\varphi_{\max}$ at which this will happen.

Part B. Two pipes *(4 points)*

An apparatus called the "Kelvin water dropper" consists of two pipes, each identical to the one described in Part A, connected via a T-junction, see fig. The ends of both pipes are at the centres of two cylindrical electrodes (with height L and diameter D with $L \gg D \gg r$). For both tubes, the dripping rate is n droplets per unit time. Droplets fall from height H into conductive bowls underneath the nozzles, cross-connected to the electrodes as shown in the diagram. The electrodes are connected via a capacitance C. There is no net charge on the system of bowls and electrodes. Note that the top water container is earthed as shown. The first droplet to fall will have some microscopic charge which will cause an imbalance between the two sides and a small charge separation across the capacitor.



i. (1.2 pts) Express the absolute value of the charge Q_0 of the drops as they separate from the tubes, and at the instant when the capacitor's charge is q. Express Q_0 in terms of $r_{\max}$ (from Part A-i) and neglect the effect described in Part A-iii.

ii. (1.5 pts) Find the dependence of q on time t by approximating it with a continuous function $q(t)$ and assuming that $q(0) = q_0$.

iii. (1.3 pts) The dropper's functioning can be hindered by the effect shown in Part A-iii. In addition, a limit $U_{\max}$ to the achievable potential between the electrodes is set by the electrostatic push between a droplet and the bowl beneath it. Find $U_{\max}$.

Problem T3. Protostar formation *(9 points)*

Let us model the formation of a star as follows. A spherical cloud of sparse interstellar gas, initially at rest, starts to collapse due to its own gravity. The initial radius of the ball is r_0 and the mass is m. The temperature of the surroundings (much sparser than the gas) and the initial temperature of the gas is uniformly T_0. The gas may be assumed to be ideal. The average molar mass of the gas is μ and its adiabatic index is $\gamma > \frac{4}{3}$. Assume that $G\frac{m\mu}{r_0} \gg RT_0$, where R is the gas constant and G is the gravitational constant.

i. (0.8 pts) During much of the collapse, the gas is so transparent that any heat generated is immediately radiated away, i.e. the ball stays in thermodynamic equilibrium with its surroundings. What is the number of times, n, by which the pressure increases when the radius is halved to $r_1 = 0.5r_0$? Assume that the gas density remains uniform.

ii. (1 pt) Estimate the time t_2 needed for the radius to shrink from r_0 to $r_2 = 0.95r_0$. Neglect the change of the gravity field at the position of a falling gas particle.

iii. (2.5 pts) Assuming that the pressure remains negligible, find the time $t_{r\to 0}$ needed for the ball to collapse from r_0 down to a much smaller radius, using Kepler's Laws.

iv. (1.7 pts) At some radius $r_3 \ll r_0$, the gas becomes dense enough to be opaque to the heat radiation. Calculate the amount of heat Q radiated away during the collapse from the radius r_0 down to r_3.

v. (1 pt) For radii smaller than r_3 you may neglect heat loss due to radiation. Determine how the temperature T of the ball depends on its radius for $r < r_3$.

vi. (2 pts) Eventually we cannot neglect the effect of the pressure on the dynamics of the gas and the collapse stops at $r = r_4$ (with $r_4 \ll r_3$). However, the radiation loss can still be neglected and the temperature is not yet high enough to ignite nuclear fusion. The pressure of such a protostar is not uniform anymore, but rough estimates with inaccurate numerical prefactors can still be done. *Estimate* the final radius r_4 and the respective temperature T_4.